DOCUMENT RESUME

ED 188 939 SE 031 463

TITLE	Biomedical Mathematics, Unit II: Propagation of Error, Vectors and Linear Programming. Instructor's Manual. Revised Version, 1975.
INSTITUTION	Biomedical Interdisciplinary Curriculum Project, Berkeley, Calif.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	75
NOTE	118p.: For related documents, see SE 031 461-471, ED 174 409-430, and ED 174 446-452. Contains occasional light and broken type.
EDRS PRICE	MF01/PC05 Plus Postage.
DESCRIPTORS	*Allied Health Occupations Education; Career Education; *Integrated Curriculum; Linear Programing; *Mathematics Curriculum; Science Course Improvement Projects; Science Education; *Secondary Education; *Teaching Guides
IDENTIFIERS	*Biomedical Interdisciplinary Curriculum Project

ABSTRACT

This instructor's manual presents lesson plans for a unit of mathematics within the Biomedical Interdisciplinary Curriculum Project (BICP), a two-year interdisciplinary precollege curriculum aimed at preparing high school students for entry into college and vocational programs leading to a career in the health field. Lessons concentrate on biomedical applications in the mathematics areas of propagation of error, vectors and linear programing. Designed to accompany the student text, lesson plans include objectives, recommended teaching time, and remarks. Keys to problem sets are also included. (CS)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

U.S. DEPARTMENT OF HEALTH,
EDUCATION & WELFARE
NATIONAL INSTITUTE OF
EDUCATION

THIS DOCUMENT HAS BEEN REPRODUCED EXACTLY AS RECEIVED FROM THE PERSON OR ORGANIZATION ORIGINATING IT. POINTS OF VIEW OR OPINIONS STATED DO NOT NECESSARILY REPRESENT OFFICIAL NATIONAL INSTITUTE OF EDUCATION POSITION OR POLICY

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Mary L. Charles
of the NSF
TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

ED188939

SE 031 463

BIOMEDICAL MATHEMATICS

UNIT II

PROPAGATION OF ERROR, VECTORS AND LINEAR PROGRAMMING

INSTRUCTOR'S MANUAL
REVISED VERSION, 1975

THE BIOMEDICAL INTERDISCIPLINARY CURRICULUM PROJECT

SUPPORTED BY THE NATIONAL SCIENCE FOUNDATION

Any opinions, findings, conclusions or recommendations expressed herein are those of the author and do not necessarily reflect the views of the National Science Foundation

Copyright © California Committee on Regional Medical Programs, 1975

TABLE OF CONTENTS

LESSON NUMBER		PAGE
1	Sources of Error and Propagation of Error Under The Operation of Addition	1
2	Propagation of Error Under The Operation of Subtraction	3
3	Propagation of Error Under The Operation of Multiplication	5
4	Relative Uncertainty	7
5	Errors That Can Be Corrected	9
6	Propagation of Error Under The Operation of Division	11
7	Relative Uncertainty Under The Operations of Multiplication And Division	13
8	Review	16
9	Introduction to Vectors and Scalar Multiplication	18
10	Vector Addition	21
11	Vector Subtraction and Vectors In N-Space	26
12	Dot Product	28
13	Dot Product II	32
14	Vector Equations--Case One	34
15	Vector Equations--Case Two	36
16	Vector Equations--Case Three	38
17	The Addition-Subtraction Method	40
18	Review	42
19	Graphical Representation of Linear Inequalities	44
20	The Region of Feasible Solutions	46
21	Linear Programming	49
22	Review	62
23	Introduction to Statistics	69
24	Tally Ho!	72
25	A Descriptive-Frequency Histogram	74
26	Yet Another Frequency Histogram	76
27	Frequency Distributions From A Theoretical Point of View	80
28	The Chi-Square Procedure	84
29	Chi Square And Your Biomed Class	87
30	How Odd (Statistically) Is Your Class?	89
31	Review	91
32	Drugs and The Placebo Effect	97

TABLE OF CONTENTS (CONT'D)

LESSON NUMBER		PAGE
33	The Cola Activity	99
34	Chi Square Problem Set	105
X	Correlation	108
Y	Uncertainties In the Vitamin C Analysis	111
Z	A Sample Ascorbic Acid Uncertainty Calculation	113

LESSON 1: SOURCES OF ERROR AND PROPAGATION OF ERROR UNDER THE OPERATION OF ADDITION

OBJECTIVES:

The student will determine the sum of two or more ranges of uncertainty.

The student will perform arithmetic operations on polynomials.

PERIODS RECOMMENDED:

One (You may need more time if the students have trouble with operations on polynomials.)

OVERVIEW AND REMARKS:

Lessons 1 through 8 of Unit II concern error and the propagation of error under the four arithmetic operations. Before teaching this material it is suggested that you reread Section 3 of the Unit I Science Text and Section 8 of the Unit I Math Text.

The earlier sections revealed that error is an inherent part of any continuous measurement procedure. In a laboratory experiment, error orignates because of imprecision in measurement and inaccuracies in measuring instruments. Then each time an arithmetic operation is performed on a number that is not accurate, the error is passed on to the number that results from the operations. This process, called propagation of error, is the main focus of the next eight lessons.

In Mathematics Lesson 8 of Unit I, measurement error is discussed in terms of ranges of imprecision. These have the form (midpoint) ± (imprecision). In the lessons to come you will find that we have departed from this language, using ranges of uncertainty instead. We wish to treat many sources of error simultaneously and therefore have replaced the narrow term "imprecision" with the more general term "uncertainty."

Ideally, there would be a thorough error analysis associated with each science laboratory. Time does not allow this, but error considerations will be discussed intermittently. You may need to assist the science teacher in preparing for those occasions.

The present lesson includes a review of operations with polynomials. Some extra time on this subject may be needed if the students have trouble. The purpose of the review is to prepare the students for the algebraic derivations in this sequence.

KEY--PROBLEM SET 1:

1. measurement error, statistical variation, procedural error and theoretical error.

2. a. 7.02 m
   b. 6.98 m
   c. 7.00 m
   d. 7 ± .02 m

3. a. 81 g
   b. 79 g
   c. 80 ± 1 g

4. (95 ± 4)%

5. 73.7 ± .45

6. 1.741 ± .021

7. 21,169,000 ± 11,500

8. .189893 ± .000794

9. a. 937
   b. 5
   c. 463
   d. 21

10. $-x^2 - 8xy - y^2$

11. $-6z + 2y - 11\Delta y$

12. $-2x^3 + 2x^2 - 12x$

13. $7m^2 - 4mn + 6n^2$

14. $\frac{13}{12}x + \frac{11}{8}\Delta x - \frac{3}{4}z$

15. $-2x + 2y + 4z$

16. $3x^3 - x^2y + xy^2 - 3y^3$

17. $x^2 + 2x - 2$

18. $x^2 + x + 8$

19. $-5\Delta x$

20. $2x^2 + 7x + 3$

21. $a^2b^2 - 9$

22. $x^4 - 7x^2y + 10y^2$

23. $a^3 + 3a^2 + 3a + 1$

24. $x^3 - 3x^2y + 3xy^2 - y^3$

LESSON 2: PROPAGATION OF ERROR UNDER THE OPERATION OF SUBTRACTION

OBJECTIVES:

The student will determine the implied uncertainty of a number or a list of numbers.

The student will round a number to a specified degree of accuracy (nearest tenth, hundredth, etc.)

The student will determine the difference of two ranges of uncertainty.

PERIODS RECOMMENDED:

Two

REFERENCE:

Nutritive Value of Foods. Home and Garden Bulletin No. 72, U.S. Department of Agriculture, Washington D.C., 1971.

OVERVIEW AND REMARKS:

The table at the beginning of the text section and the one in the problem set were taken from the reference cited. Such tables will be used extensively in the upcoming vector and linear programming sequence (Lessons 9 through 22).

KEY--PROBLEM SET 2:

1. .5 g

2. .05 mg

2. .5 mg

4. 250 international units

5. .5

6. .0005

7. .005

8. .5 or 5

9. .5, 5, 50 or 500

10. 6 g

11. 8 m

12. 7.4 mg

13. 6.20 cm

14. a. 35

    b. 29

    c. 32

    d. 3

    e. 32 ± 3

15. c

16. c

17. 2.7 ± .7

18. -3.9 ± .9

19. 4.2 ± .9

20. 1.9 ± 1.7

21. 6.1 ± 4.6

22. 1.6 ± .6

23. 15.350 ± .010 g

24. 1450 ± .015 g

LESSON 3: PROPAGATION OF ERROR UNDER THE OPERATION OF MULTIPLICATION

OBJECTIVES:

When given two specific ranges of uncertainty, the student will compute the maximum product, minimum product, midpoint of the product and uncertainty of the product.

The student will use the relations $(x \pm \Delta x)(y \pm \Delta y) = (xy + \Delta x\Delta y) \pm (x\Delta y + y\Delta x)$ and $(x \pm \Delta x)(y \pm \Delta y) \approx xy \pm (x\Delta y + y\Delta x)$ to compute products of ranges of uncertainty.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The derivation of the literal formula for the product of two ranges of uncertainty is included in the text, but does not play a part in the problem set or subsequent material. Depending on your class, you may wish to omit the derivation.

KEY--PROBLEM SET 3:

1. a. 9.2
   b. 2.8
   c. 6
   d. 3.2
2. a. $g + \Delta t$
   b. $g - \Delta t$
   c. g
   d. Δt
3. a. 63
   b. 25
   c. 44
   d. $\frac{63 - 25}{2} = \frac{38}{2} = 19$
   e. 44 ± 19
4. a. 1
   b. 2
   c. 2
   d. no
   e. The uncertainty of the product is $x\Delta y + y\Delta x$
5. a. $a = 10, \Delta a = 1$
   $b = 15, \Delta b = 2$
   b. midpoint, uncertainty
   c. $(ab + \Delta a\Delta b) \pm (a\Delta b + b\Delta a) = (10 \times 15 + 1 \times 2) \pm (10 \times 2 + 15 \times 1)$
   $= (150 + 2) \pm (20 + 15)$
   $= 152 \pm 35$
6. a. 132 ± 34 (approximate)
   134 ± 34 (exact)
   b. 180 ± 1.18 (approximate)
   180.001 ± 1.18 (exact)
   c. 225 ± .90 (approximate)
   225.0005 ± .90 (exact)
   d. $hs \pm (h\Delta s + s\Delta h)$ (approximate)
   $(hs + \Delta h\Delta s) \pm (h\Delta s + s\Delta h)$ (exact)
7. $(70 + \frac{1}{150}) \pm \frac{41}{30}\ m^2$ (exact)
   $70 \pm \frac{41}{30}\ m^2$ (approximate)
8. a. .005
   b. .005
   c. 99.9996, .00005
   d. $(9.98 \times .005 + 10.02 \times .005)$
   $= .005(9.98 + 10.02)$
   $= .005\ (20)$
   $= .1$

LESSON 4: RELATIVE UNCERTAINTY

OBJECTIVE:

The student will interconvert absolute uncertainty and relative uncertainty.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

Problem 16 of the problem set involves much written explanation. In a class with reading difficulties, you might consider discussing the problem in class with students acting out the roles of Alfred and Elmo.

KEY--PROBLEM SET 4:

1. a. .1 calorie
   b. 1%
   c. 10 ± 1% calories
2. a. 2%
   b. 3 meters
   c. 150 ± 3 meters
3. 5 ± 2% seconds
4. 20 ± .5% calories
5. 100 ± .5% meters
6. 8.8 ± .088 calories
7. 5.67 ± .567 grams
8. 25.25 ± .12625 calories
9. a. 10 ± .3
   b. 10 ± 3%
10. a. 10.0 ± .21
    b. 10.0 ± 2.1%
11. a. .2, .3
    b. (20 ± .2) - (15 ± .3)
    c. 5 ± .5
    d. 5 ± 10%
12. a. 30
    b. 1
    c. .3
13. a. 28
    b. 7
    c. 25%
14. a. 80.48 ml   c. 125.75 ml
    b. 45.27 ml   d. 125.75 ml
15. a. .04 ml
    b. 40 ± .04 ml
16. a. Alfred
    b. Elmo
    c. ~.7874% ≃ .8%
    d. .60 cm

LESSON 5: ERRORS THAT CAN BE CORRECTED

OBJECTIVE:

The student will correct measurements that possess an estimable relative error.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 5:

1. a. absolute

   b. 111 lb

2. a. relative

   b. ~350 lb

3. $\frac{1}{11}(60) \approx 5.5$ mph

   $60 - 5.5 = 54.5$ mph

4. a. $5.2 - 5.0 = .2$ mi

   b. $\frac{.2}{5.2} \approx .03846$

   $\approx 3.8\%$

   c. $\frac{.2}{5.2} \cdot 130 = 5$ mi

   d. more, subtracted

   e. $130 - 5 = 125$ mi

   f. $\frac{.2}{5.2} \cdot 65 = 2.5$ mph

   g. $65 - 2.5 = 62.5$ mph

5. a.

$$\begin{array}{r} 35029.9 \text{ mi} \\ -35026.2 \text{ mi} \\ \hline 3.7 \text{ mi} \end{array}$$

   b. $4 - 3.7 = .3$ mi

   c. $\frac{.3}{3.7} \approx .08108$

   $\approx 8.1\%$

   d. $\frac{.3}{3.7} \cdot 125 \approx 10.1$ mi

   ≈ 10 mi

   $125 + 10 = 135$ mi

   e. The indicated distance is less than the actual distance.

   f. $.081 \cdot 55 = 4.455$

   ≈ 4.5

   $55 + 4.5 = 59.5$

6. a. 197 mi

   b. 56 mph

7. a. (1) 15% is wrong. Instead of a relative error of $\frac{.3}{2}$ it should have been $\frac{.3}{1.7}$ or about 18%

   (2) Elmo subtracted the correction factor instead of adding it.

   b. Who knows?

LESSON 6: PROPAGATION OF ERROR UNDER THE OPERATION OF DIVISION

OBJECTIVE:

When given two specific ranges of uncertainty, the student will compute the maximum quotient, minimum quotient, midpoint of the quotient and uncertainty of the quotient.

The student will use the relations $\frac{x \pm \Delta x}{y \pm \Delta y} = \frac{xy + \Delta x \Delta y}{y^2 - \Delta y^2} \pm \frac{x\Delta y + y\Delta x}{y^2 - \Delta y^2}$ and $\frac{x \pm \Delta x}{y \pm \Delta y} \approx \frac{x}{y} \pm \frac{x\Delta y + y\Delta x}{y^2}$ to compute the quotients of ranges of uncertainty.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 6:

1. 22

2. $\frac{1998 - 180}{100 + 1}$

3. 18

4. midpoint $= \frac{22 + 18}{2}$

   $= 20$

5. absolute uncertainty $= \frac{22 - 18}{2}$

   $= 2$

6. 20 ± 2

7. a. 19.98 ± 1.9998

   b. yes

8. a. $\frac{6 \pm .5}{50 \pm .5}$

   b. $\frac{300 + .25}{2500 - .25} \pm \frac{3 + 25}{2500 - .25}$

   c. $\frac{300}{2500} \pm \frac{28}{2500}$

9. $20 \pm \frac{4 + 2}{4} = 20 \pm 1.5$

10. $.6 \pm \frac{1.2 + 1}{100} = .6 \pm .022$

11. $.6 \pm \frac{.3 + .2}{100} = .6 \pm .005$

12. $.402 \pm \frac{.0402 + .1}{100} \simeq .402 \pm .001$

13. $.78 \pm \frac{.156 + .8}{400} \simeq .780 \pm .002$

14. $.363 \pm \frac{.0726 + .2}{100} \simeq .363 \pm .003$

15. $.32 \pm \frac{8 + 25}{2500} \simeq .32 \pm .01$

16. $.625 \pm \frac{.1275 + .204}{46.24} \simeq .625 \pm .007$

LESSON 7: RELATIVE UNCERTAINTY UNDER THE OPERATIONS OF MULTIPLICATION AND DIVISION

OBJECTIVES:

The student will compute relative uncertainties for given absolute ranges of uncertainty.

The student will provide the terms necessary to complete sentences of the following types. "The relative uncertainty of a product is approximately the ______ of the relative uncertainties of the factors." "The relative uncertainty of a quotient is approximately the _____ of the relative uncertainties of numerator and denominator."

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

This lesson consists of a problem set which is intended to be completed during the class period. The problem set uses previous skills (error propagation, interconversion of absolute and relative uncertainty) in order to guide the students toward the conclusion that relative uncertainty is additive under the operations of multiplications and division. Your main role in this lesson will lie in providing assistance as the students work the problem set.

KEY--PROBLEM SET 7:

1. a. ml solution, $\frac{\text{units antitoxin}}{\text{ml solution}}$, units antitoxin

   b. $(100 \pm 1)(1500 \pm 30) \simeq (1500)(100) \pm (100 \times 30 + 1500 \times 1)$

   $\simeq 150{,}000 \pm (3000 + 1500)$

   $\simeq 150{,}000 \pm 4500$

   c. $\frac{1}{100} \times 100\% = 1\%$

   d. $\frac{30}{1500} \times 100\% = 2\%$

   e. $\frac{4500}{150{,}000} \times 100\% = 3\%$

   f. 1% + 2% = 3%

2. a. ml solution, $\frac{\text{mg terramycin}}{\text{ml solution}}$, mg terramycin

   b. $(1.5 \pm .015)(200 \pm 1) \simeq (1.5)(200) \pm (1.5 \times 1 + .015 \times 200)$

   $\simeq 300 \pm (1.5 + 3)$

   $\simeq 300 \pm 4.5$

   c. $\frac{.015}{1.5} \times 100\% = \frac{1.5}{1.5}\%$

   $= 1\%$

   d. $\frac{1}{200} \times 100\% = \frac{1}{2}\%$

   $= .5\%$

   e. $\frac{4.5}{300} \times 100\% = 1.5\%$

   f. 1% + .5% = 1.5%

3. a. 3%

   b. 3%

   c. 1.5%

   d. 1.5%

   e. factors, product

4. a. $\frac{\Delta x}{x} \cdot 100\%, \ \frac{\Delta y}{y} \cdot 100\%$

   b. $\frac{x\Delta y + y\Delta x}{xy} \cdot 100\%$

   c. $x\Delta y + y\Delta x$

   d. relative

   e. relative

5. a. $\frac{2499 + 75}{100 - 1} = \frac{2574}{99}$

   $= 26$

   b. $\frac{2499 - 75}{100 + 1} = \frac{2424}{101}$

   $= 24$

   c. $\frac{26 + 24}{2} = \frac{50}{2}$

   midpoint = 25

   $\frac{26 - 24}{2} = \frac{2}{2}$

   absolute uncertainty = 1

   d. $\frac{2499}{100} = 24.99$

   e. $\frac{75}{2499} \simeq \frac{75}{2500}$

   $\frac{75}{2500} \cdot 100\% = 3\%$

   f. $\frac{1}{100} \cdot 100\% = 1\%$

   g. $\frac{1}{25} \cdot 100\% = 4\%$

   h. sum, relative

6. Yes

LESSON 8: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 1 through 7.

PERIODS RECOMMENED:

One

KEY--REVIEW PROBLEM SET 8:

1. a. 63 ± .11
   b. .04
   c. .07
   d. .11
   e. sum

2. a. 633 ± 6
   b. 4
   c. 2
   d. 6
   e. sum

3. a. 50,000 ± 1500
   b. 2
   c. 5
   d. 1500
   e. 2%
   f. 1%
   g. 3%
   h. sum

4. a. $30 \pm \left(\frac{300 \cdot 1 + 10 \cdot 3}{100}\right) = 30 \pm 3.3$
   b. 3
   c. 1
   d. 3.3
   e. 1%
   f. 10%
   g. 11%
   h. sum

5. a. 1500 ± 30 meters
   b. 6.02 ± .602 grams or
      6.02 ± .6 grams

6. a. 0.5
   b. 0.005
   c. one of 0.5, 5, 50, 500
   d. 0.00005
   e. 0.0005

7. a. 900 ± 60 miles
   b. 9 ± 1 trips
   c. $\frac{900 + 60}{9 - 1} \, \frac{\text{miles}}{\text{trip}} = 120 \, \frac{\text{miles}}{\text{trip}}$
   d. $\frac{900 - 60}{9 + 1} \, \frac{\text{miles}}{\text{trip}} = 84 \, \frac{\text{miles}}{\text{trip}}$
   e. $102 \pm 18 \, \frac{\text{miles}}{\text{trip}}$

LESSON 9: INTRODUCTION TO VECTORS AND SCALAR MULTIPLICATION

OBJECTIVES:

The student will identify row vectors, column vectors, and matrices.

The student will express food components in row vector notation and interpret such notation when given.

The student will perform scalar multiplication of vectors.

PERIODS RECOMMENDED:

One

REFERENCES:

Bowes, Anna de P. and Charles F. Church. Food Values of Portions Commonly Used. Latest edition, J.B. Lippincott, Philadelphia.

Nutritive Value of Foods. Home and Garden Bulletin No. 72, U.S. Dept. of Agriculture, Washington, 1971.

OVERVIEW AND REMARKS:

You are now beginning a sequence of fourteen lessons on vectors and linear programming. The application of this sequence centers on the nutritive contents of foods. Vectors provide a convenient tool for the analysis of food tables. A given amount of a food can be represented by a vector, with components representing certain nutrients, for example protein, fat and carbohydrate. Vector addition, subtraction and dot product are developed in this setting. Finally, linear programming is applied to the problem of finding the cheapest combination of foods meeting given nutritional requirements. Such problems are central in trying to feed the world's population.

The U.S. Dept. of Agriculture Bulletin listed above was used as the primary source on nutritive contents. This publication will also be used in the science course. Prior to the beginning of Science Unit II, sets adequate for class use should be available through the science instructor.

The Bowes and Church reference serves as a supplementary source. It contains a more extensive analysis than the U.S.D.A. source. The science instructor will have a couple of copies of this publication.

Science Unit II is devoted to nutrition and there are several relevant (although not necessarily concurrent) ties. Laboratory Activity 2 is designed to acquaint the student with the U.S.D.A. reference. In Science Lesson 3, protein, fat, carbohydrate and calories are discussed briefly.

The study of nutritional chemistry, which is most relevant to this math sequence, begins in Science Lesson 10.

KEY--PROBLEM SET 9:

1. a, b, c, e

2.

row vector	e
column vector	a or b
matrix	f
scalar	d or g

3. There is an arrow over the R, and there are square brackets enclosing numbers separated by commas.

4. 31
   8
   11

5. 1 g protein
   3 g fat
   14 g carbohydrate

6. 4

7. a. 14
      16
      2

   b. [7, 8, 1]

   c. (4)

8. a. 6 g protein
      3 g fat
      36 g carbohydrate

   b. [2, 1, 12]

   c. [6, 3, 36]

9. b

10. a. [18, 18, 24]

    b. [64, 64, 8]

11. a. $12\vec{S} = [60, 12, 72]$

    b. no

12. $\frac{1}{4}$

13. a. $20\vec{F} = [40, 20, 720]$

    b. protein, carbohydrate

    c. fat

    d. 64

14. $24\frac{3}{8}$ g

LESSON 10: VECTOR ADDITION

OBJECTIVES:

The student will determine whether or not two vectors are equivalent.

The student will interpret graphical representations of vectors in two-space.

The student will graphically represent vectors in two-space.

The student will perform vector addition in two-space by graphical methods and in n-space by analytical methods.

PERIODS RECOMMENDED:

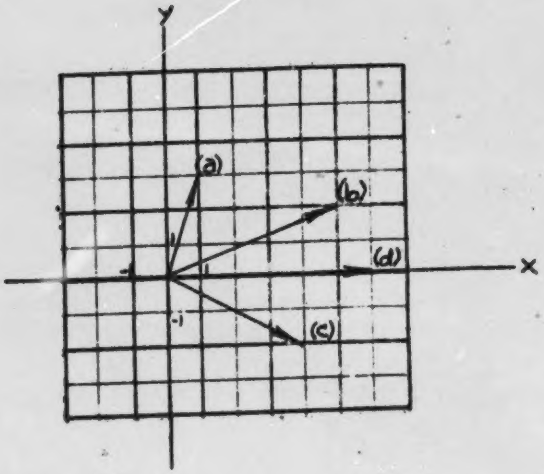
One

OVERVIEW AND REMARKS:

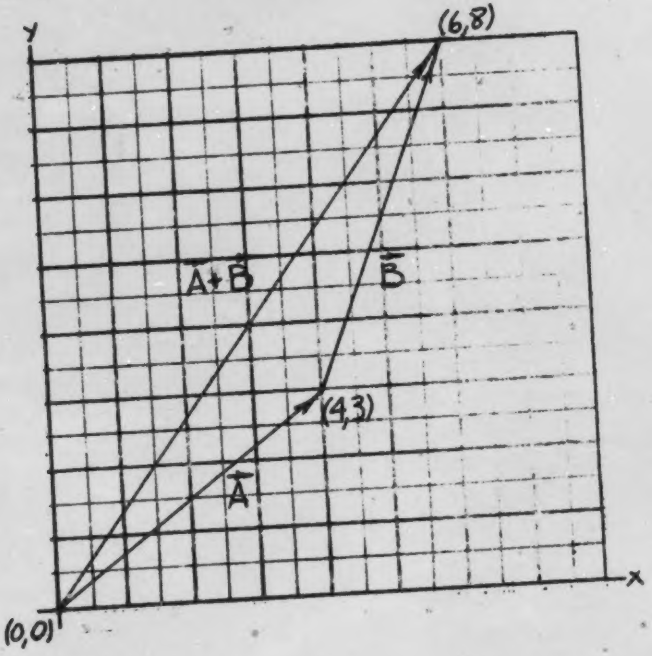
In this lesson, vectors are interpreted graphically. The idea that a food, for example, can be represented by an arrow on a graph will probably take some convincing. The graphical approach to vector problems will continue in subsequent lessons.

KEY--PROBLEM SET 10:

1. a. $\vec{B}$
   b. (4, 2)
   c. (0, 0)
   d. $\vec{C}$
   e. $\vec{A}$
   f. equivalent
2. a. [3, 4]
   b. [-3, 4]
      [-3, -4]
   c. $\vec{F}$
   d. $\vec{D}$
   e. $\vec{C}$
3.


y
x
(a)
(b)
(c)
(d)
1
-1
1
-1

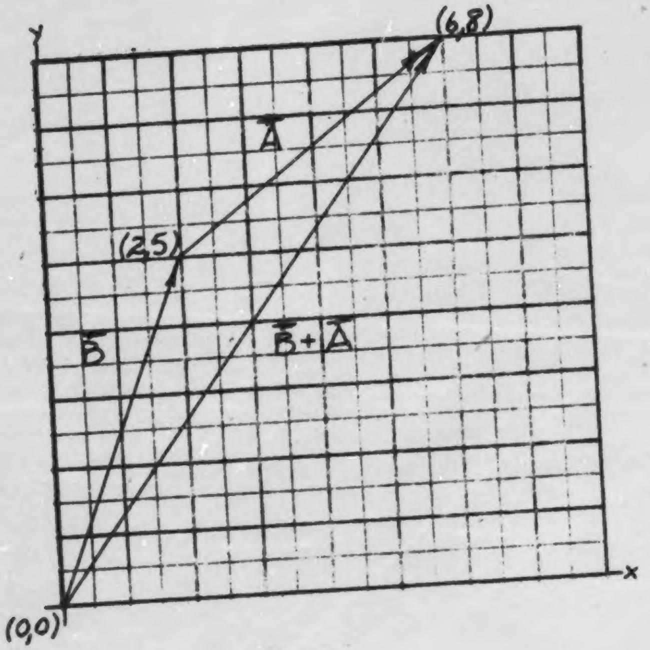
4. a and b



y
(6,8)
$\overline{A}+\overline{B}$
$\overline{B}$
(4,3)
$\overline{A}$
(0,0)
x

c. (1)

d.



y
(6,8)
$\overline{A}$
(2,5)
$\overline{B}$
$\overline{B}+\overline{A}$
(0,0)
x

e. yes

f. yes

g. yes

5. b.

6. c

7. a. [10, 13]

   b. [17, 3]

   c. [4, 1]

   d. [-3, -2]

   e. [a + 2, b + 3]

   f. [16, 14]

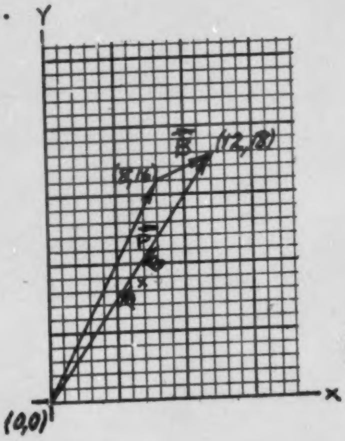
   g. [6, 13]

   h. [7, 13, 3]

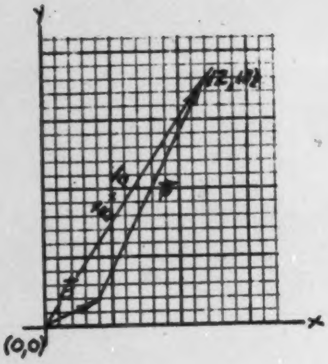
   i. [3, 6, 8]

   j. [18, 12, 6]

8. a.

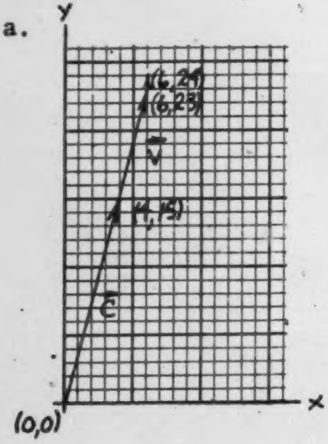


b.

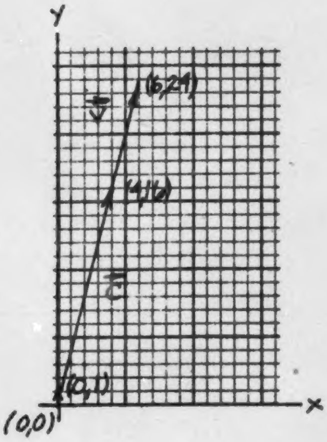


c. yes

9. a.



b.



c. [6, 24]

c. yes

10. [11, 9, 60]

11. 2.7 mg (with 2 slices of bread)

12. white bread

13. rye bread

14. $$\begin{bmatrix} .25 \\ 0 \\ .06 \\ .06 \\ .04 \end{bmatrix}$$

15. [2, 1, 13]

16. [8, 8, 1, 262, .3, 320, 0, 0, 0]

17. [34, 28, 25, 308, 4.3, 320]

18. a. One slice of ham and 2 slices of cheese on two slices of french bread.

b. [31, 28, 28]

c. fat

LESSON 11: VECTOR SUBTRACTION AND VECTORS IN N-SPACE

OBJECTIVES:

The student will perform vector subtraction by analytical methods.

The student will determine the additive identity and additive inverse of a given vector.

The student will interpret graphical representations of vectors in three-space.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The treatment of three-space vectors is brief; you may want to expand some of the ideas. Problem 29 is a good discussion problem.

KEY--PROBLEM SET 11:

1. a. [12, 8]
    b. [-2, 7, -1]
2. c
3. d
4. [0, 0, 0]
5. b
6. zero
7. [-5, -6, -7]
8. [-1, 0, -10]
9. [2, -5, 6]
10. [100, 50, 1]
11. c
12. c
13. a
14. a
15. no
16. a. [3, 11]
    b. [1, 8]
    c. [-11, -7]
    d. [-3, 13]
    e. [3, 3, 3]
    f. [a - x, b - y, c - z]
    g. [20, 2]
    h. [11, 9]
17. a. $2\vec{R} = [6, -2, 4]$
    b. $3\vec{S} = [-6, 15, -9]$
    c. $2\vec{R} - 3\vec{S} = [12, -17, 13]$
18. [-10, 31, -27]
19. [-30, 9, -18]
20. [5, -5, 3]
21. [-11, 4, -1]
22. [28.5, 29.5, 35.5]
23. a. [27, 32, 28]
    b. $\vec{X} = [8, 1, 10]$
24. $\vec{X} = [23, 14, 0]$
25. $\vec{X} = [4, 1, 32]$
26. [1, 0, 14]
27. 2 stalks
28. a. 3
    b. (3)
29. yes

    4 - dimensional space.

LESSON 12: DOT PRODUCT

OBJECTIVES:

The student will calculate the dot product of two vectors.

The student will interpret and use summation notation.

PERIODS RECOMMENDED:

The combination of this lesson and Problem Set 13 should probably be given three periods. If you wish to give a test after Lesson 13, you may want to spend more time and include a review in your discussion of the problem sets.

OVERVIEW AND REMARKS:

Care will need to be taken to avoid confusion between the dot product and scalar multiplication. You may also encounter some difficulty with percentages if students are weak in this area.

The concept of calories appears here for the first time. You may want to discuss this idea. Calories are explained briefly in Science Lesson 3 and more thoroughly in Science Lesson 22.

KEY--PROBLEM SET 12:

1. d
2. b
3. -2
4. 5
5. $x + y$
6. x
7. y
8. c
9. 10
10. 6
11. 0
12. z
13. $-x -y -z$
14. $x_1y_1 + x_2y_2 + x_3y_3 + x_4y_4 + x_5y_5 + x_6y_6$
15. 51 cal
16. 13 acorns
17. 63,200 people
18. no
19. false
20. $2[x, y, z] = [2x, 2y, 2z]$, which is a vector

    $[2, 2, 2]\cdot[x, y, z] = 2x + 2y + 2z$, which is a scalar
21. a.
22. $\sum_{i=2}^{5} x_i$
23. $\sum_{i=1}^{100} s_i$

24. $\sum_{i=1}^{n} a_i b_i$

25. $\sum_{i=1}^{6} x_i y_i$

26. b.

27. $\vec{K} = [4, 9, 4]$

28. a. .24 g protein, .32 g fat, 0 g carbohydrate

b. 24 g protein, 32 g fat, 0 g carbohydrate

c. $\vec{S} = [24, 32, 0]$

d. $[4, 9, 4] \cdot [24, 32, 0] = 96 + 288 + 0$

$= 384$ cal

29. a. 2 g protein, 6 g fat, 17.5 carbohydrate

$\vec{F} = [2, 6, 17.5]$

b. $\vec{K} \cdot \vec{F} = [4, 9, 4] \cdot [2, 6, 17.5] = 8 + 54 + 70 = 132$ cal

30. a. $\vec{T} + \vec{L} + \vec{D} = [4, 8, 8]$

b. $\vec{K} \cdot \vec{S} = 120$ cal

31. a. $\vec{K} \cdot \vec{M} = 165$ cal

b. $\vec{K} \cdot \vec{C} = 468$ cal

c. $\vec{M} + \vec{C} = [14, 29, 79]$

$\vec{K} \cdot (\vec{M} + \vec{C}) = 633$ cal

d. yes

32. 120 cal - salad
384 cal - steak
132 cal - French fries
633 cal - milk and chocolate cake

1269 cal total
+ 2000 cal

3269 cal
- 3000 cal

269 cal total

Yes--he will have consumed 269 extra calories.

*33. a. $\vec{W} = [.4, 1.0, .6]$

b. $\vec{M} = [36, 36, 48]$

c. $\vec{W} \cdot \vec{M} \approx 14 + 36 + 29 = 79$ g

d. $976 - (36 + 36 + 48) = 856$ g

e. $856 + 79 = 935$ g

f. Water lost to heat dissipation = $640 \times .6 = 384$ g

Water lost in wastes = $.8 \times 976 \approx 781$ g

Total water lost ≈ 1165 g

deficit $\approx 935 - 1165 = 230$ g

g. thirstier

*34. a. $\vec{O}_2 = [.9 \frac{\text{liter}}{\text{g}}, 1.9 \frac{\text{liters}}{\text{g}}, .8 \frac{\text{liter}}{\text{g}}]$

$[.9 \frac{\text{liter}}{\text{g}}, 1.9 \frac{\text{liters}}{\text{g}}, .8 \frac{\text{liter}}{\text{g}}] \cdot [10 \text{ g}, 20 \text{ g}, 55 \text{ g}] =$

$9 + 38 + 44 = 91$ liters

b. $[.9 \frac{\text{liter}}{\text{g}}, 1.9 \frac{\text{liters}}{\text{g}}, .8 \frac{\text{liter}}{\text{g}}] \cdot [36 \text{ g}, 36 \text{ g}, 48 \text{ g}]$

$= 32.4 + 68.4 + 38.4$

$= 139.2$ liters

LESSON 13: DOT PRODUCT II

OBJECTIVE:

The student will calculate the dot product of two vectors.

The student will compute the calories used in performing various activities for specified periods of time.

PERIODS RECOMMENDED:

See Lesson Plan 12.

OVERVIEW AND REMARKS:

There is no text for this lesson, only a problem set. You may want to demonstrate Problem 2b.

KEY--PROBLEM SET 13:

1. a. $\vec{K} = [4, 9, 4]$

   $\vec{F} = [6, 14, 1]$

   $\vec{K} \cdot \vec{F} = 154$ calories

   b. 130 calories

   c. 284 calories

2. a. Mass in pounds = X

   $$X \text{ pounds} \cdot \frac{.454 \text{ kg}}{1 \text{ pound}} = X \cdot (.454) \text{ kg}$$

   b. Activity uses Y cal/kg-min

   Mass = X kg

   Activity = X kg • Y cal/kg-min = X • Y cal/min

3. There are 284 calories in one hot dog, from Problem 1.
   Thus 2 hot dogs have 568 calories.
   If student's mass is K kilograms, he uses K(.025) cal/min.
   It takes him 8 min, so he uses 8 min • K • (.025)cal/min = (.2)• K cal.
   568 calories - (.2)• K calories = net calories gained after eating.

4. Activity requires M cal/kg-min.
   Student's mass = K.
   Student's activity uses K kg • M cal/kg-min = K • M cal/min.
   He has 568 - .2K calories left.

   $$\text{Time} = \frac{568 - .2\text{K calories}}{\text{K} \cdot \text{M calories/min}}$$

* 5. $.025K - \frac{284}{4} = 0$

   K = 2,840 kg

6. a. Dinner has 1199 calories

   b. Student's mass = K kg
   Eating costs .025 cal/kg-min
   K kg •(.025)cal/kg-min • 40 min = K calories

6. c. Student's mass = K kg
   Activity requires M cal/kg-min = K • cal/min
   Student has 1199 - K calories left.

   $$\text{Time} = \frac{1199 - \text{K calories}}{\text{K} \cdot \text{M calories/min}}$$

LESSON 14: VECTOR EQUATIONS--CASE ONE

OBJECTIVE:

The student will solve vector equations of the form $x[a, o] + y[o, b] = [c, d]$.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 14:

1. b
2. a. no
   b. yes
   c. no
   d. yes
   e. yes
3. $x = 16$, $y = 10$
4. $x = r$, $y = s$, $z = t$
5. b, c and f
6. a
7. b
8. $x = 3$, $y = 4$
9. $x = 1$, $y = 3$
10. $x = 30$, $y = 5$
11. 300 g apples and 170 g salmon
12. a. $\vec{H} = [18.48, 0]$
    b. $\vec{B} = [0, 22.5]$
    c. $\vec{D} = [37, 67.5]$
    d. approximately 2 ham slices and 3 bananas
13. a. pork chops: $\vec{A}$ or $x\vec{A}$
    cherries: $\vec{B}$ or $y\vec{B}$
    diet: $\vec{C}$
    b. 21 g fat and 9 g carbohydrate
    c. 100 g pork chops and 60 g cherries, or approximately 1 pork chop and $\frac{1}{2}$ cup cherries.

LESSON 15: VECTOR EQUATIONS--CASE TWO

OBJECTIVE:

The student will solve vector equations of the form x[a, b] + y[c, d] = [e, f], when one of the components a, b, c or d is equal to zero.

PERIODS RECOMMENDED:

One and one-half or two.

OVERVIEW AND REMARKS:

The idea of representing a nutritional problem graphically may cause difficulty. Some explanation will probably be needed.

KEY--PROBLEM SET 15:

1. vectors

2. equivalent

3. a (3)

   b. (2)

   c. $x = 2, \quad y = 3$

4. $4x$

5. $6y$

6. $3y$

7. $10x + 7y$

8. $5x$

9. $6x + y = 20$

   $5y = 2$

10. $7x = 5$

    $6x + 2y = 15$

11. $100x + 18y = 18$

    $12x = 0$

12. $x = 25, \quad y = 10$

13. $x = 4, \quad y = 2$

14. $x = 1, \quad y = 5$

15. $x = 0, \quad y = 3$

16. $x = 2, \quad y = 4$

17. a. $\vec{J} = [61, 14]$

    $\vec{C} = [0, 5]$

    $\vec{D} = [122, 33]$

    b. (3)

    c. $x = 2, \quad y = 1$

    d. 2 glasses of orange juice, 1 bowl of corn flakes

18. 2 spoonfuls of peanut butter, 3 spoonfuls of jelly

19. 25 strawberries, 2 scoops of ice cream

20. 3 slices of bread and 3 tablespoons of honey, or 1 tablespoon of honey per slice of bread

21. 1 egg, 2 cups of milk

22. 10 bananas, 200 g of hamburger

23. 25 tablespoons of peanut butter, $\frac{3}{8}$ cup of sugar

24. 3 sections of pizza, 1 glass of cola
    504 calories

LESSON 16: VECTOR EQUATIONS--CASE THREE

OBJECTIVE:

The student will solve vector equations of the form x[a, b] + y[c, d] = [e, f], using the substitution method.

PERIODS RECOMMENDED:

One and one-half or two.

KEY--PROBLEM SET 16:

1. a. $x = 2, y = 9$
   b. $x = 8, y = 3$
   c. $x = 20, y = 80$
   d. $x = 5, y = 6$
   e. $x = 2, y = 1$

2. $x[5500, 4] + y[100, 4] = [5900, 20]$

   4 stalks of celery, 1 carrot

3. 11 apples, 14 pears

4. 2 cups macaroni and $\frac{1}{2}$ cup cheese

   4 : 1

5. 1 sandwich

   3 glasses of milk

6. $3x + y = 1$

   $6x + 5y = 3$

   $x = \frac{2}{9}$ head of lettuce

   $y = \frac{1}{3}$ carrot

   2 heads of lettuce and 3 carrots

44

LESSON 17: THE ADDITION-SUBTRACTION METHOD

OBJECTIVE:

The student will solve vector equations of the form x[a, b] + y[c, d] = [e, f], using the addition-subtraction method.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 17:

1. $x = 3$, $y = 5$

2. $x = 40$, $y = 70$

3. $x = 100$, $y = 300$

4. $x = 100$, $y = -100$

5. $x = 150$, $y = 210$

6. $x = 200$, $y = 500$

7. a, b and c

8. a. chicken: .325 g of protein per g

   .013 g of carbohydrate per g

   milk: .039 g of protein per g

   .052 g of carbohydrate per g

   b. $\vec{C} = [.325, .013]$

   $\vec{M} = [.039, .052]$

   c. Note that all of the numerical values are divisible by 13. The resulting equations are

$$25x + 3y = 59{,}000 \quad \text{and} \quad x + 4y = 13{,}000.$$

$$x \approx 2{,}031 \text{ g of chicken}$$

$$y = 2{,}742 \text{ g of milk}$$

   d. $[4, 9, 4] \cdot [767, 239, 169] = 3068 + 2151 + 676 = 5{,}895$ calories

9. $\vec{D} = [39, 93]$

   $\vec{A} = [15, 51]$ per piece

   $\vec{I} = [.12, .21]$ per gram

   One piece of apple pie and 200 g of ice cream

LESSON 18: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 9 through 17.

PERIODS RECOMMENDED:

Two

OVERVIEW AND REMARKS:

As part of this review you could discuss Science Worksheet 2 in the context of vectors. The worksheet is based on the U.S.D.A. source.

KEY--PROBLEM SET 18:

1. [5, -3, -8]
2. [-3, -16, 11]
3. [1, 21, -4]
4. $2\vec{H} + \vec{B} + \frac{1}{2}\vec{R}$
5. [46, 16, 29]
6. 444 calories
7. [4, 2, 8]
8. 292 calories
9. 52.5 calories
10. 231 calories
11. 8.5 calories
12. 56 minutes
13. a. $x = 3$ $y = 4$

    b. $x = -4$ $y = 6$

    c. $x = 9$ $y = 6$
14. $x = 7$ $y = 6\frac{1}{2}$
15. 90 g tuna and 80 g tangerines.
16. 2 drumsticks and $1\frac{1}{2}$ helpings of salad.
17. 120 g steak and 75 g French fries

LESSON 19: GRAPHICAL REPRESENTATION OF LINEAR INEQUALITIES

OBJECTIVES:

The student will identify true, false and open inequalities.

The student will translate statements into inequalities.

The student will match a set of inequalities with their graphical representation.

The student will represent a set of inequalities graphically.

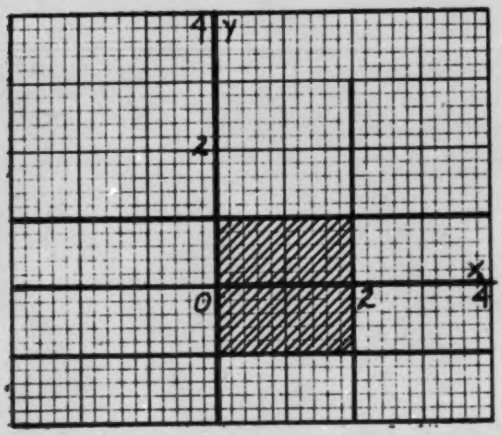
PERIODS RECOMMENDED:

Two. This allows time for a review of graphing linear functions.

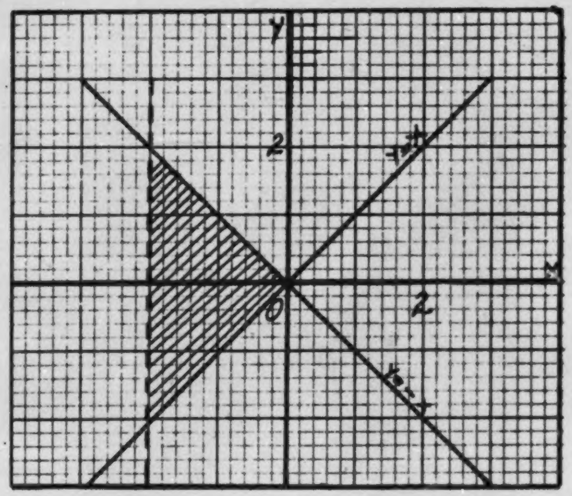
KEY--PROBLEM SET 19:

1. a. True
   b. False
   c. False
   d. True
   e. Open
   f. Open
   g. False
   h. Open
   i. True
2. a. $5 < 6$
   b. $-3 \geq -10$
   c. $2 < 3 < 5$
      or $3 > 2$ and $3 < 5$
   d. $s < 0$
3. a. w
   b. u
   c. z
   d. y
   e. x
   f. v
4. a. z
   b. v
   c. y

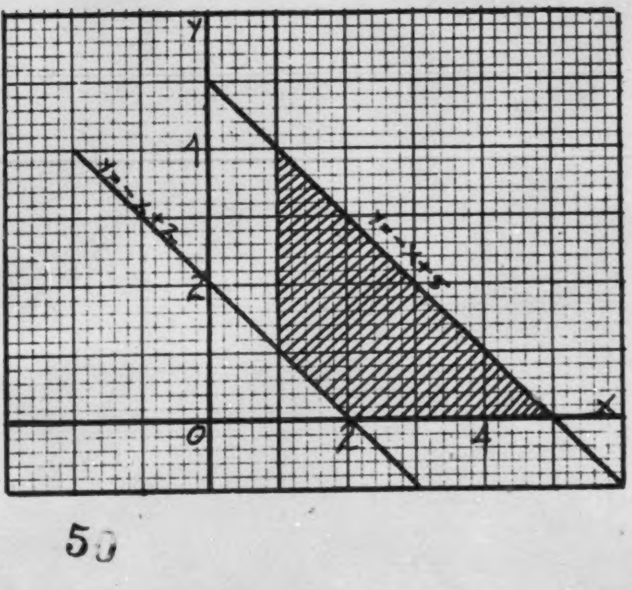
5.



6.



7.



LESSON 20: THE REGION OF FEASIBLE SOLUTIONS

OBJECTIVES:

The student will determine a system of linear inequalities from a vector inequality of the form $x\vec{W} + y\vec{S} \geq \vec{D}$.

The student will transform the system of linear inequalities into standard $y \geq mx + b$ form.

The student will determine graphically the "region of feasible solutions" for a system of linear inequalities.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 20:

1. $y > 4$ or $4 < y$

2. $-4 \le x$ or $x \ge -4$

3. $s \le -9$ or $-9 \ge s$

4. $q \ge -6$ or $-6 \le q$

5. $y \ge \frac{1}{5}x - \frac{7}{5}$

6. $y \ge -\frac{1}{2}x + \frac{15}{2}$

7. $y \ge -\frac{1}{2}x - 2$

8. $y \ge -2x + 14$

9. a. $x[2, 3, 4] + y[1, 8, 7] \ge [5, 10, 15]$

   b. $[2x, 3x, 4x] + [y, 8y, 7y] \ge [5, 10, 15]$

   c. $[2x + y, 3x + 8y, 4x + 7y] \ge [5, 10, 15]$

   d. $2x + y \ge 5$

   $3x + 8y \ge 10$

   $4x + 7y \ge 15$

10. a. $x[2, 4] + y[3, 8] \ge [15, 10]$

   b. $[2x, 4x] + [3y, 8y] \ge [15, 10]$

   c. $[2x + 3y, 4x + 8y] \ge [15, 10]$

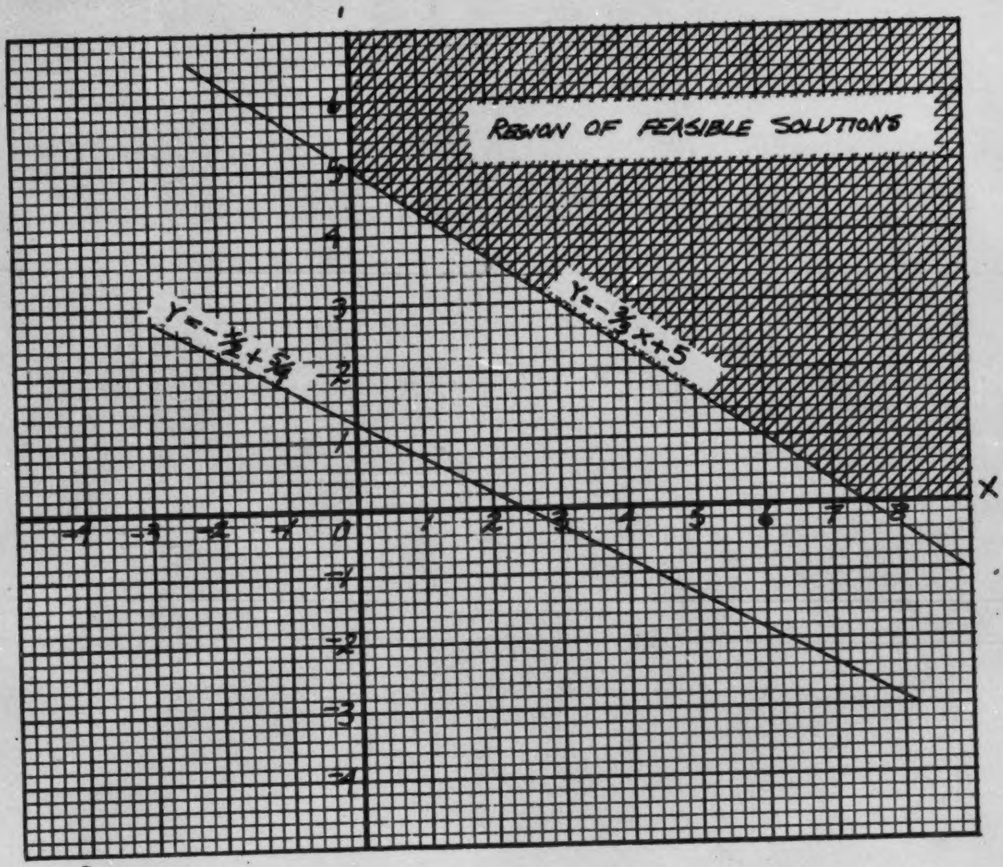
   d. $2x + 3y \ge 15$

   $4x + 8y \ge 10$

   e. $y \ge -\frac{2}{3}x + 5$

   $y \ge -\frac{1}{2}x + \frac{5}{4}$

f.

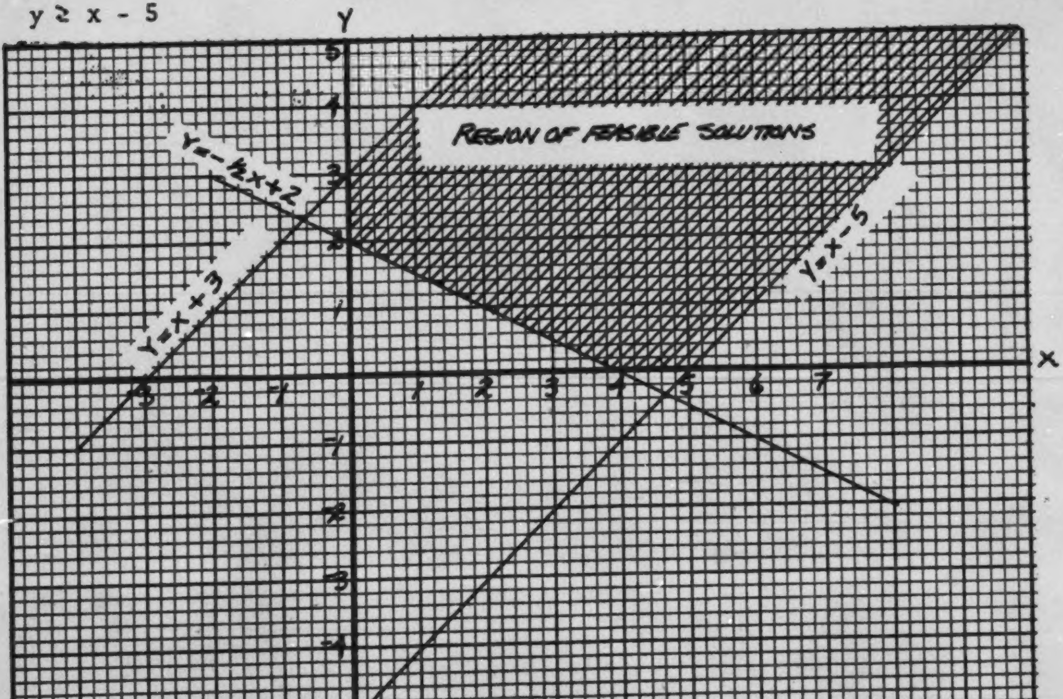


11. a. $y \geq -\frac{1}{2}x + 2$

$y \leq x + 3$

$y \geq x - 5$

b.



LESSON 21: LINEAR PROGRAMMING

OBJECTIVE:

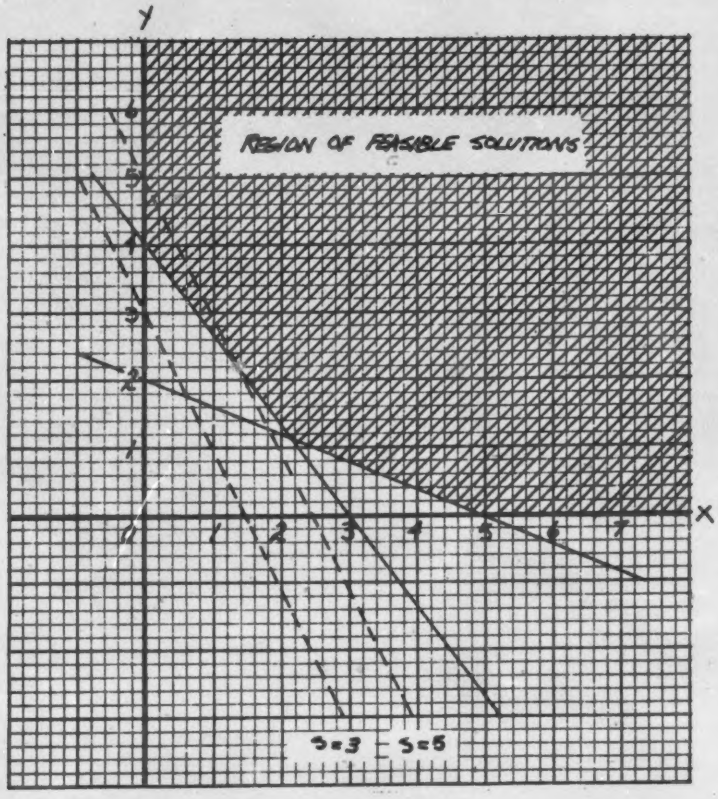
The student will solve linear programming problems which involve minimizing a linear function on a region of feasible solutions.

PERIODS RECOMMENDED:

Four or five

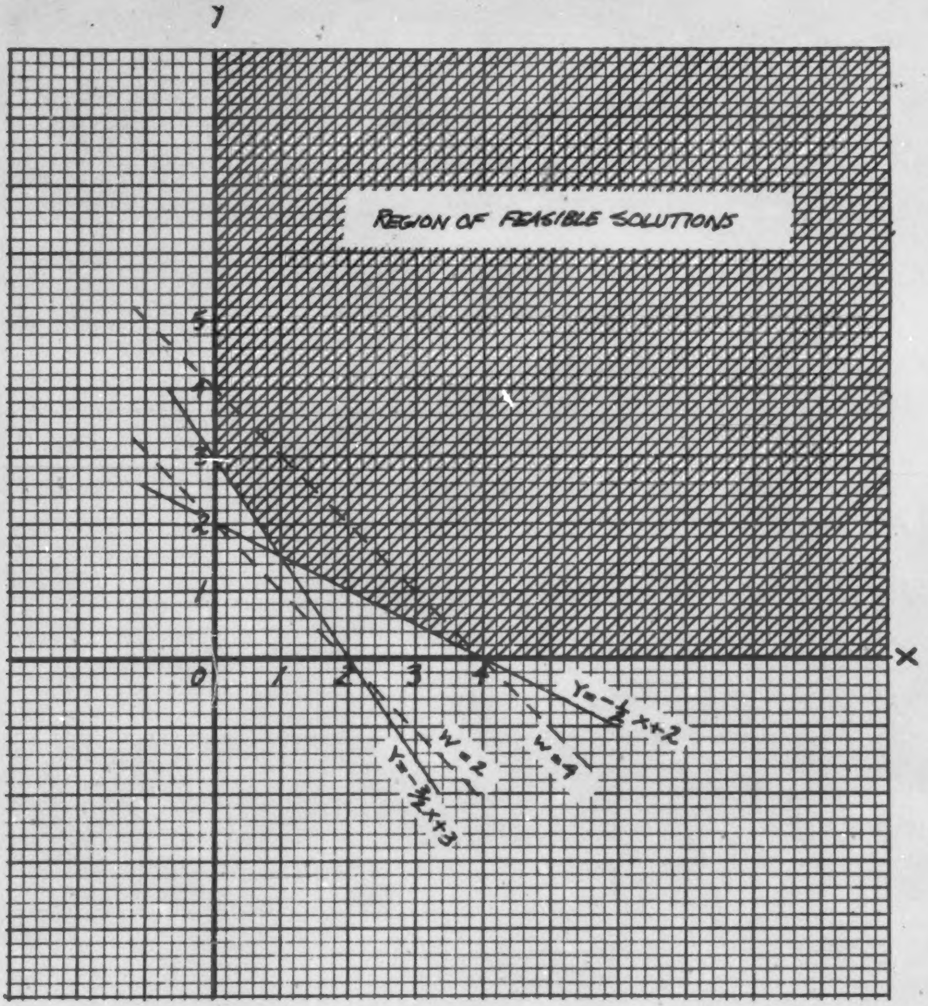
KEY--PROBLEM SET 21:

1. a. 4
   b. (1)
   c. lower
2. a. 10
   b. B
   c. (3)
3. a. $y = -2x + 5$
   b.



   c. A
   d. (0,4)
   e. 4

4. a.



b. $y = -x + w$

c. see graph

d. $x = 1$, $y = \frac{3}{2}$

e. $2\frac{1}{2}$

5. a. $x\vec{C} + y\vec{S} \geq \vec{D}$ represents the statement of the problem, namely, that whatever the number of pounds x of cabbage or y of snails, (1) the sum of the protein contents must equal or exceed that of $\vec{D}$ and (2) the sum of the carbohydrate contents must equal or exceed that of $\vec{D}$.

b. $x\vec{C} + y\vec{S} \geq \vec{D}$

$x[15, 25] + y[30, 10] \geq [60, 40]$

$[15x, 25x] + [30y, 10y] \geq [60, 40]$

$15x + 30y \geq 60$ or $x + 2y \geq 4$

$25x + 10y \geq 40$ or $5x + 2y \geq 8$

c. $x + 2y \geq 4$ $5x + 2y \geq 8$

$2y \geq -x + 4$ $2y \geq -5x + 8$

$y \geq -\frac{1}{2}x + 2$ $y \geq -\frac{5}{2}x + 4$

d. A negative number of pounds is meaningless.

e. See graph on following page.

f. $\text{Time} = 30\,\frac{\text{minutes}}{\text{lb cabbage}}\,(x \text{ lb cabbage}) + 20\,\frac{\text{minutes}}{\text{lb snails}}\,(y \text{ lb snails})$

$T = 30x + 20y$

g. $T = 30x + 20y$

$20y = -30x + T$

$y = -\frac{3}{2}x + \frac{1}{20}T$

h. see graph

i. see graph

j. $(1, \frac{3}{2})$

k. $T = 30(1) + 20(\frac{3}{2})$

$= 30 + 30$

$= 60$ minutes

6. a. true

b. similar to 5a

c. $x[90, 100] + y[15, 400] \geq [180, 800]$

$[90x, 100x] + [15y, 400y] \geq [180, 800]$

$[90x + 15y, 100x + 400y] \geq [180, 800]$

(1) $90x + 15y \geq 180$

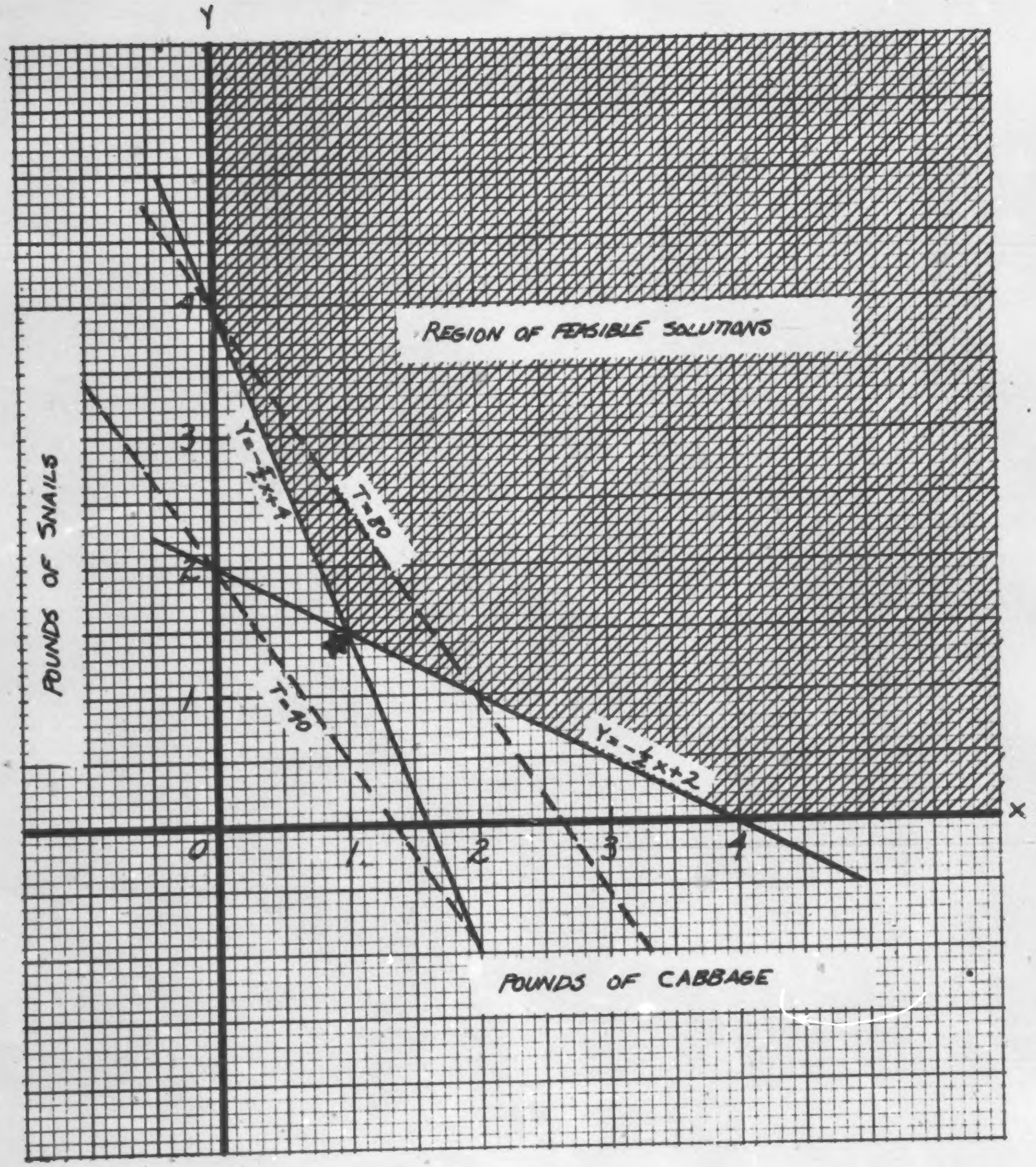
(2) $100x + 400y \geq 800$

d. (1) $90x + 15y \geq 180$

$15y \geq -90x + 180$

$y \geq -6x + 12$

e.

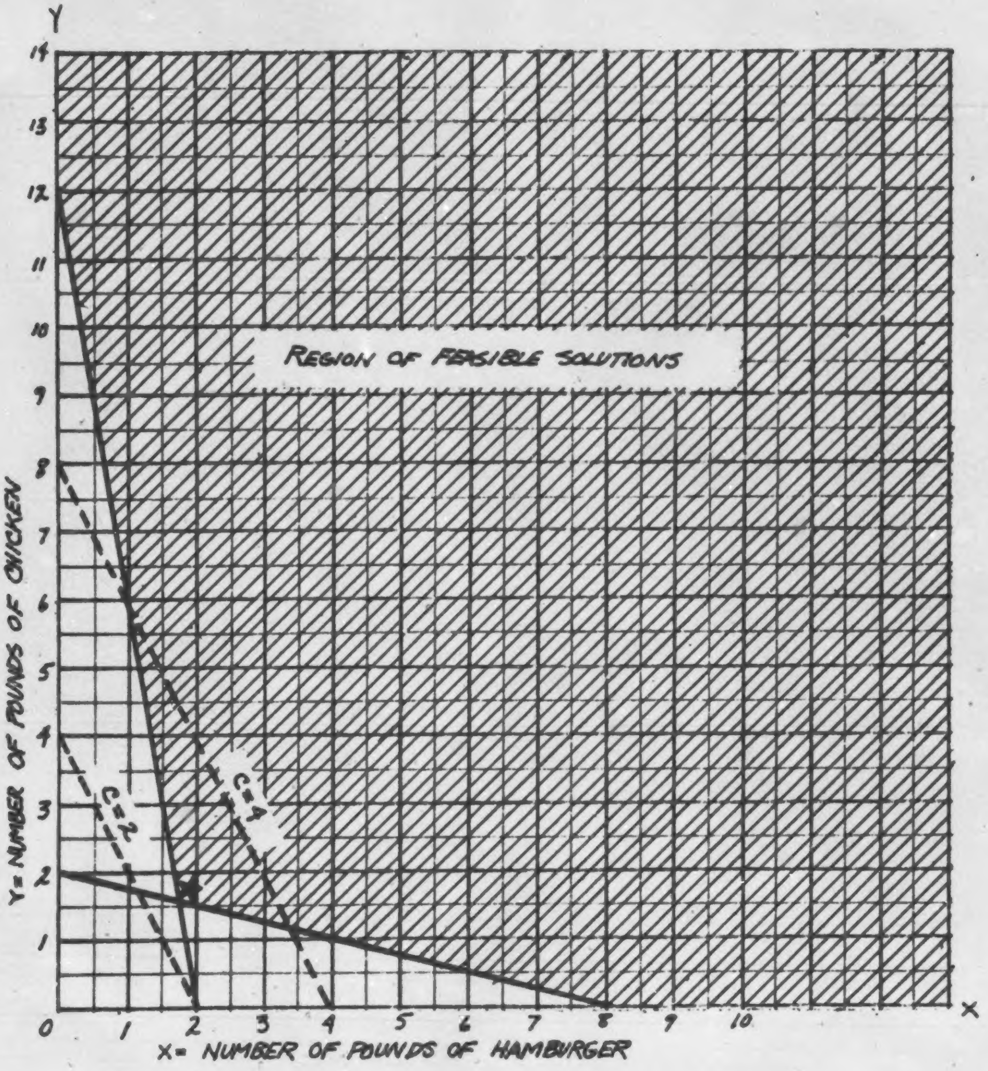

Y
X
REGION OF FEASIBLE SOLUTIONS
POUNDS OF SNAILS
POUNDS OF CABBAGE
$Y=-\frac{?}{?}x+4$
$T=80$
$T=40$
$Y=-\frac{1}{2}x+2$
0
1
2
3
4
1
2
3
4

d. (2) $100x + 400y \geq 800$

$$400y \geq -100x + 800$$

$$y \geq -\frac{1}{4}x + 2$$

e. A negative number of pounds is meaningless.



g. cost in dollars =

$$x \text{ lb hamburger } \left(\frac{1 \text{ dollar}}{1\text{b hamburger}}\right) + y \text{ lb chicken } \left(\frac{\frac{1}{2} \text{ dollar}}{1\text{b chicken}}\right)$$

cost in dollars $= x + \frac{1}{2}y$

h. $c = x + \frac{1}{2}y$

$\frac{1}{2}y = -x + c$

$y = -2x + 2c$

i. see graph for Part f.

j. see graph for Part f.

k. $x = \frac{40}{23}$, $y = \frac{36}{23}$

$\frac{40}{23}$ lb hamburger

$\frac{36}{23}$ lb chicken

l. $c \approx \$2.52$

7. a. The constraints $x \geq 0$ and $y \geq 0$ prohibit a negative number for either type of pill.

b. x blue pills plus y red pills must meet or exceed Dr. Mobucs' cure vector $\vec{C}$.

c. $x[1, 8, 6] + y[2, 5, 1] \geq [12, 74, 24]$

$[x, 8x, 6x] + [2y, 5y, y] \geq [12, 74, 24]$

$x + 2y \geq 12$

$8x + 5y \geq 74$

$6x + y \geq 24$

d. $x + 2y \geq 12$

$2y \geq -x + 12$

$y \geq -\frac{1}{2}x + 6$

$8x + 5y \geq 74$

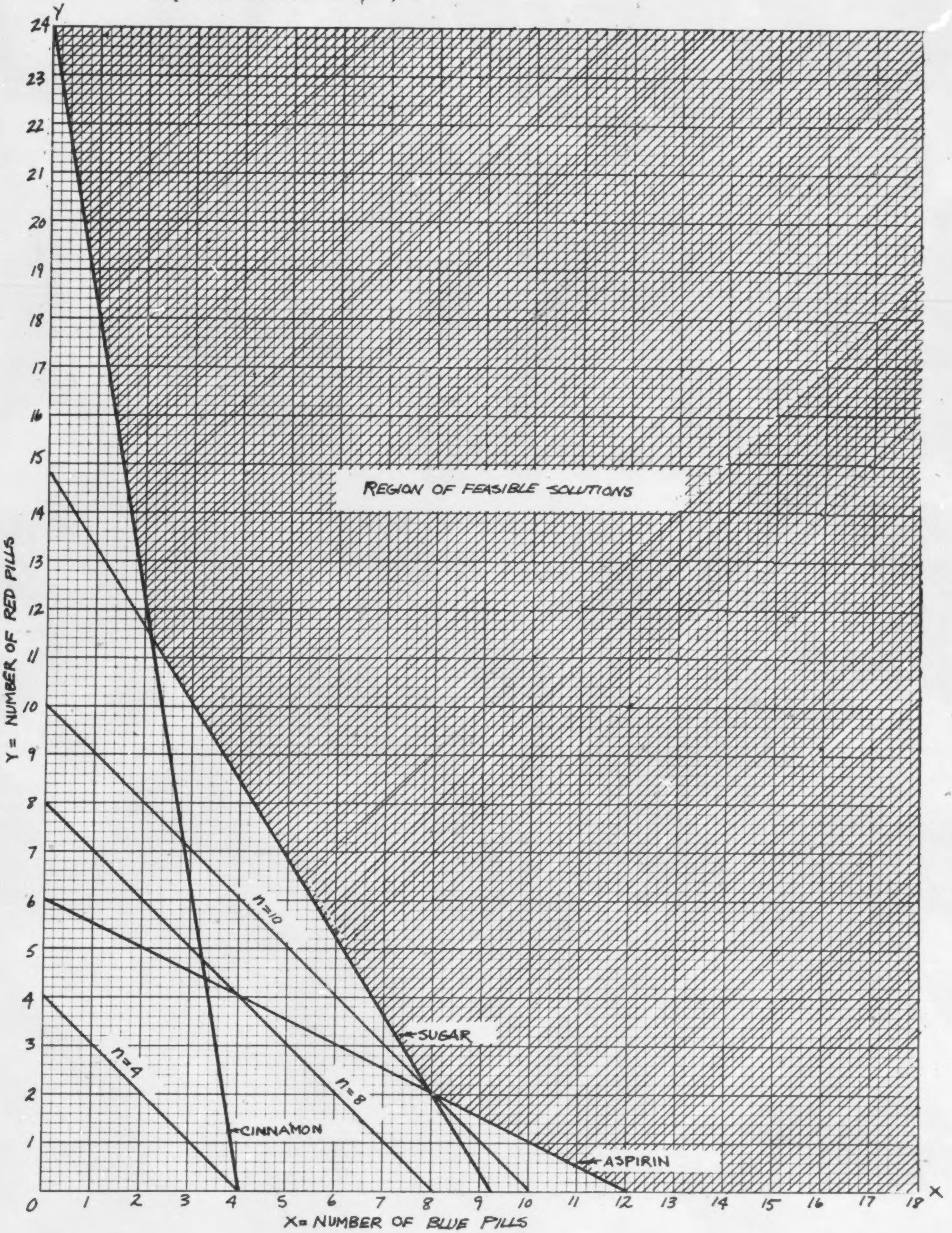
$5y \geq -8x + 74$

$y \geq -\frac{8}{5}x + \frac{74}{5}$

$6x + y \geq 24$

$y \geq -6x + 24$

Graph for Problems 7e, f, i



g. n is the sum of the number of yellow pills and the number of red pills.

h. $n = x + y$

$x + y = n$

$y = -x + n$

i. See graph on preceding page.

j. (1) $<$

(2) $<$

(3) $\leq$

k. $x = 8$

$y = 2$

$n = 10$

8. a. $\vec{B} = [1.2, 8.0, 2.4]$

$\vec{P} = [1.5, 5.0, 1.2]$

$\vec{C} = [2.0, 3.0, 1.5]$

b. (2)

c. $5.0x + 3.0y \geq 8.0$ (LYS)

$1.2x + 1.5y \geq 2.4$ (MET)

d. $y \geq -\frac{5}{3}x + \frac{8}{3}$ (LYS)

$y \geq -\frac{4}{5}x + \frac{8}{5}$ (MET)

e. (4)

f. See graph on following page.

g. (1)

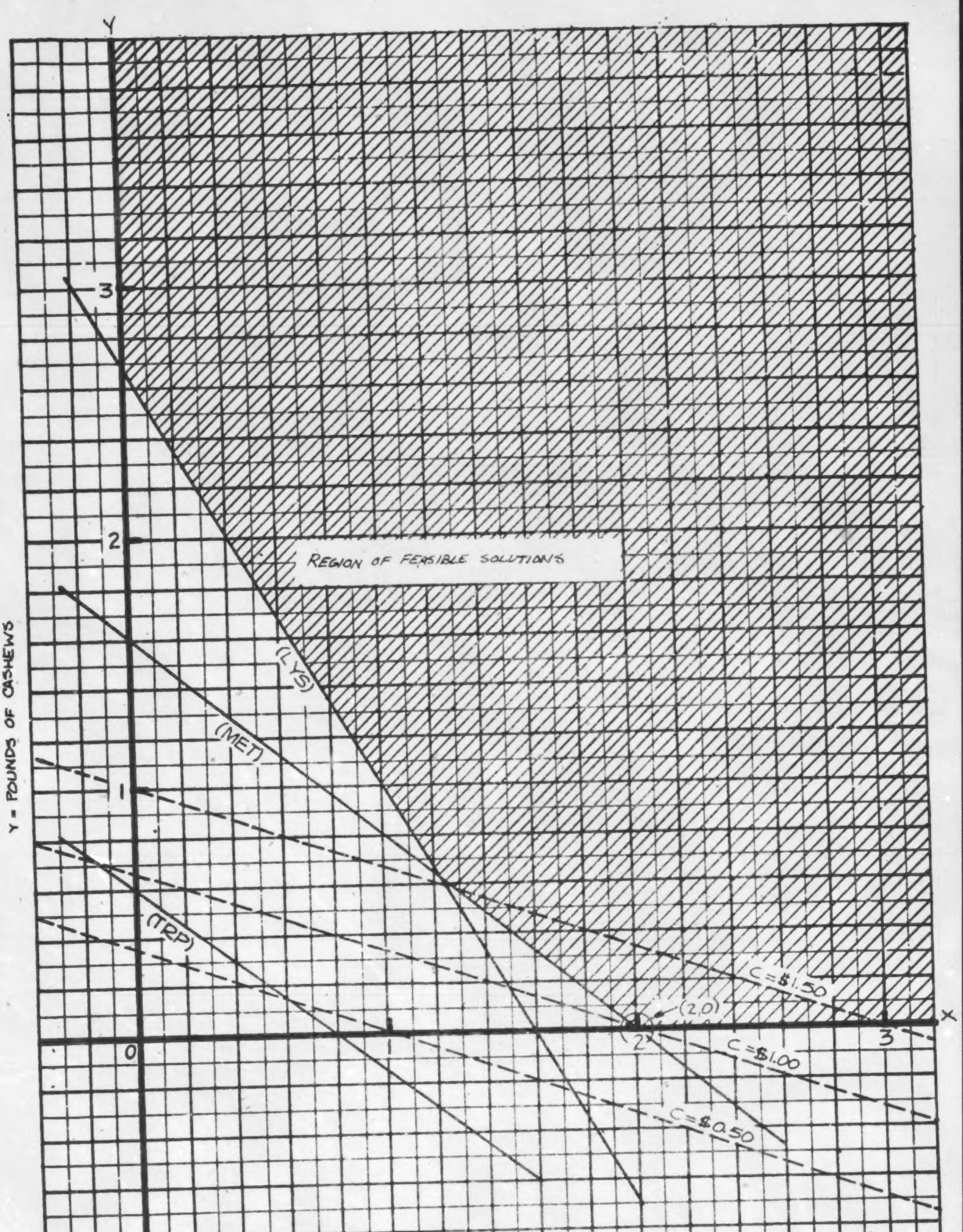
h. (2)

i. See graph on following page.

j. \$1.00

$x = 2 \quad y = 0$

2 lb peanuts and no cashews.


Y
X
3
2
1
0
1
2
3
REGION OF FEASIBLE SOLUTIONS
(LYS)
(MET)
(TRP)
C=$1.50
C=$1.00
C=$0.50
(2,0)
Y = POUNDS OF CASHEWS
X = POUNDS OF PEANUTS

9. Let x = pounds of pinto beans, y = number of eggs.

a. Write three inequalities to express the fact that the diet of beans and eggs must contain at least as much of each amino acid as a pound of beef.

$$1.0x + 0.1y \geq 1.2$$

$$7.5x + 0.4y \geq 8.0$$

$$1.0x + 0.2y \geq 2.4$$

b. Write two inequalities to indicate that a negative quantity of either beans or eggs is impossible.

$$x \geq 0$$

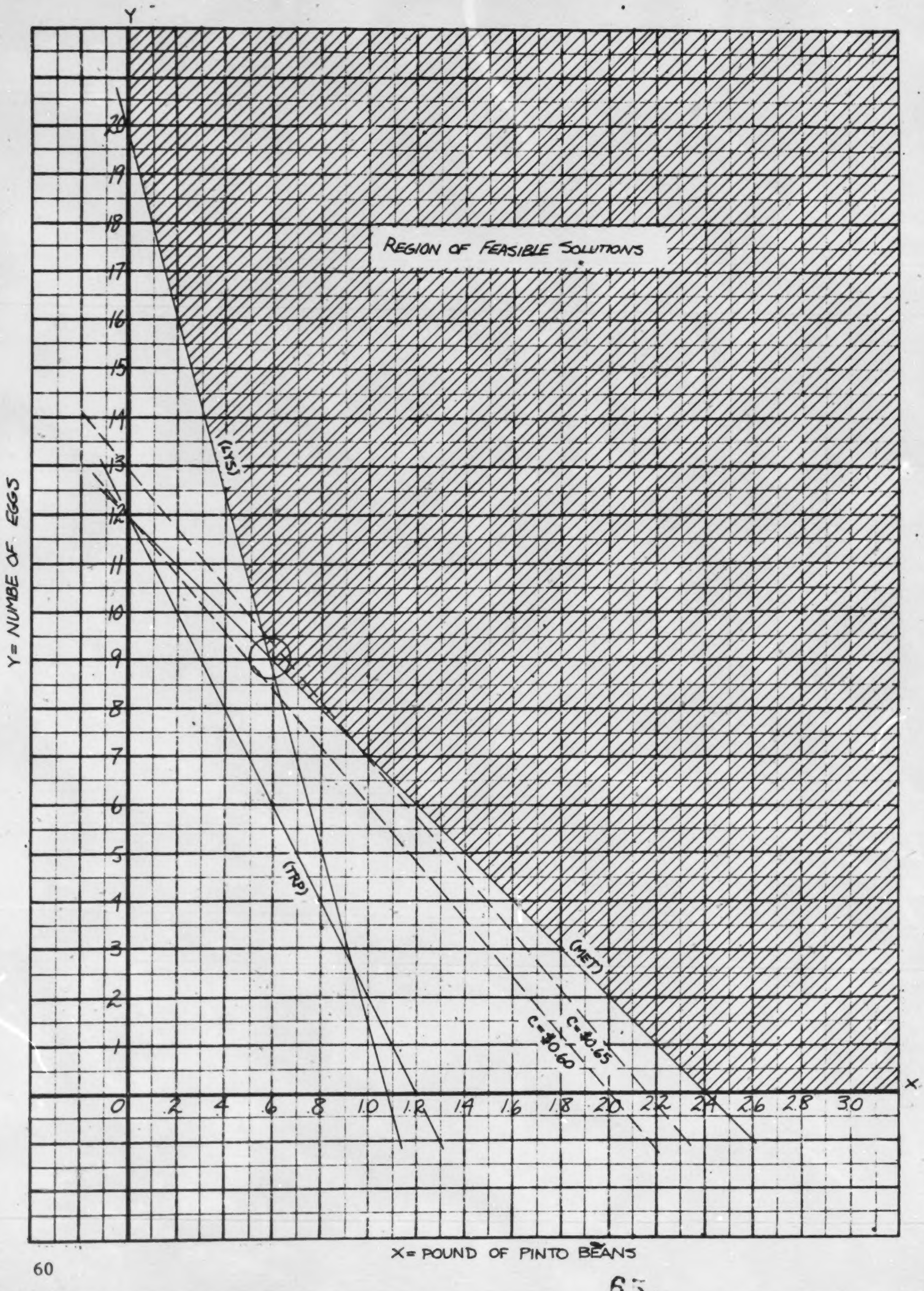
$$y \geq 0$$

c. Make a graph with pounds of pinto beans plotted against number of eggs. Graph the five inequalities of Parts a and b on the graph. Label the region of feasible solutions. (See the following graph.)

d. The cost c (in cents) of x pounds of pinto beans and y eggs is given by the equation

$$c = 30x + 5y$$

This equation is plotted on the graph for $c = 60¢$ and $c = 65¢$. The optimum solution is for $c \approx 63¢$. At this point $x \approx .6$ and $y \approx 9$.


Y
X
REGION OF FEASIBLE SOLUTIONS
(LYS)
(TRP)
(MET)
C=$0.65
C=$0.60
Y= NUMBE OF EGGS
X= POUND OF PINTO BEANS
20
19
18
17
16
15
14
13
12
11
10
9
8
7
6
5
4
3
2
1
0
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
2.2
2.4
2.6
2.8
3.0

10. Let x = the number of 100 gram lots of eggs to be allotted per person per day. Let y = the number of 100 gram lots of pancake mix to be allotted per person per day.

Then the solutions to the problems must be found among the values of x and y such that the following constraints are satisfied.

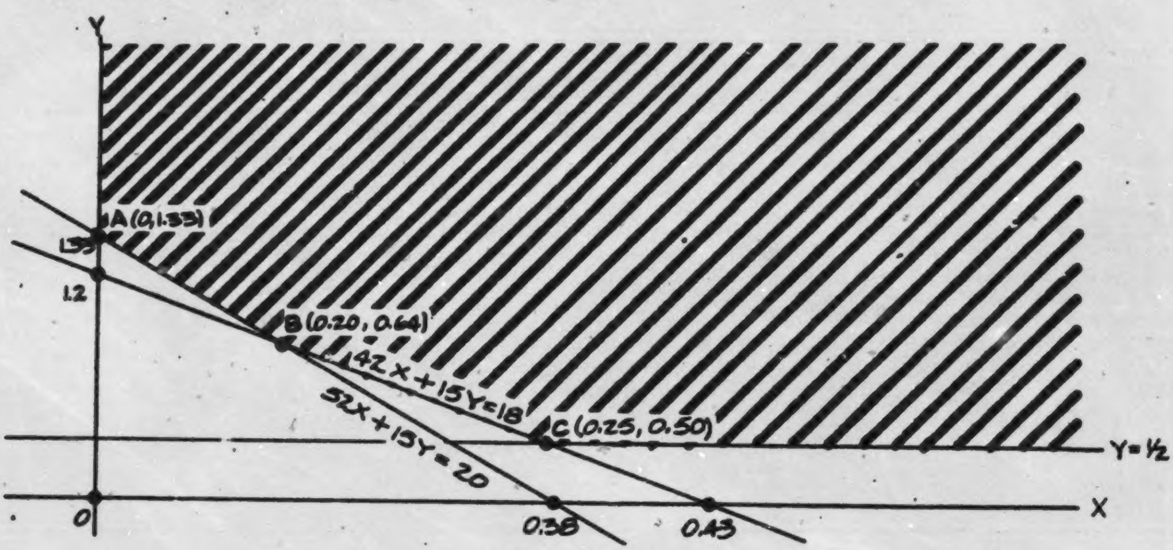
$$x \geq 0 \qquad\qquad y \geq 0$$

$$52x + 15y \geq 20$$

$$14x + 5y \geq 6$$

$$60y \geq 30 \text{ (or } y \geq \tfrac{1}{2}\text{)}$$

This defines the following shaded region.



a. 25 g dried eggs and 50 g pancake mix per person per day. (This is roughly equivalent to 2 eggs and 3 $\frac{1}{2}$ pancakes.)

b. No eggs and 133 g pancake mix. (This is roughly equivalent to 9 pancakes.)

LESSON 22: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 1-22.

PERIODS RECOMMENDED:

Two

PREPARATION FOR FUTURE LESSON:

Lesson 23 is the first in a sequence on the chi-square procedure. In that lesson, there is an activity involving coin tossing. Eight coins per student will be required and a few extras are recommended.

KEY--PROBLEM SET 22:

1. 97 ± .15
2. 1.259 ± .013
3. $(5.724 \pm .011) \times 10^6$
4. 27,900,000 ± 1500
5. a. .25
   b. .0025
6. 19.84 ± .04 g
7. 5.8 ± .2
8. 2.21 ± .04
9. -1635 ± 21
10. 17.22 ± .06
11. 300 ± 35
12. a. 100 ± 2
    b. 1000 ± 30
13. $400 \pm .8\ \text{cm}^2$
14. $1 \pm .3\ \text{m}^3$
15. 1%
16. .2%
17. .06%
18. 4.35
19. .46
20. 1.46
21. a. 210 ± .7
    b. $210 \pm \frac{1}{3}\%$
22. 20.5 mpg
23. a. 19 ± .1 mpg
    b. Yes, because his timing tinkering reduced his mpg.
24. a. $.05 \pm .00064\ \frac{\text{g}}{\text{ml}}$
    b. $5 \pm .064\ \frac{\text{g}}{100\ \text{ml}}$
25. 3.8%
26. a. $\vec{B} = [2, 1, 12]$
    b. $\vec{L} = [40, 20, 240]$
27. a. [0, 0, -1]
    b. [8, -9, -20]
28. a. [1, 1, 9]
    b. [1, 3, -11]
    c. [2, 1, 28]
29. a. 3
    b. 3
    c. yes
30. 20
31. $\vec{K} \cdot \vec{S} = [4, 9, 4] \cdot [3, 0, 7] = 40$ cal
32. a. $2x + y = 8$
       $-x - y = 11$
    b. $x = 19,\ y = -30$
33. 8 pieces of cheese
    4 apples
34. $\frac{4}{3}$ California oranges
    $\frac{14}{9}$ Florida oranges
35. $x = \frac{1}{4}$
    $y = 4\frac{3}{4}$

36. a. true c. open

b. open d. false

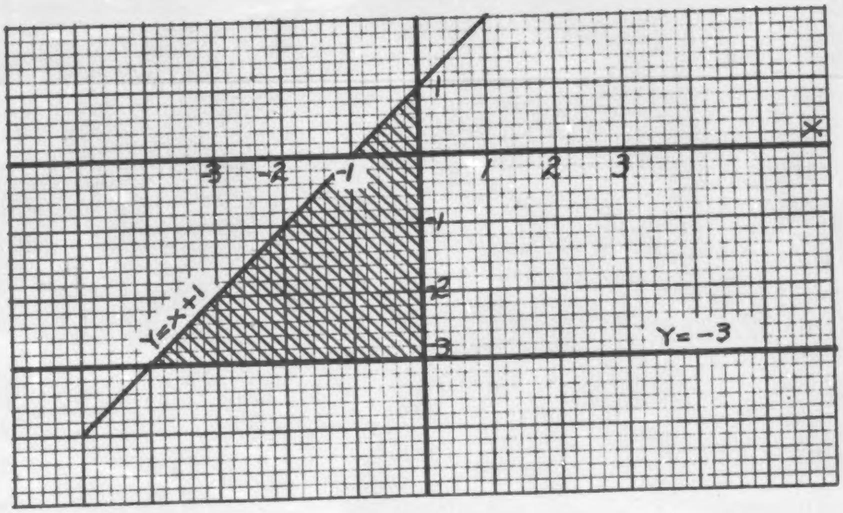
37. a. $5 > -2$ and (but) $5 < 8$

b. $2x = \frac{1}{2}x + 3$ or $2x - 3 = \frac{1}{2}x$

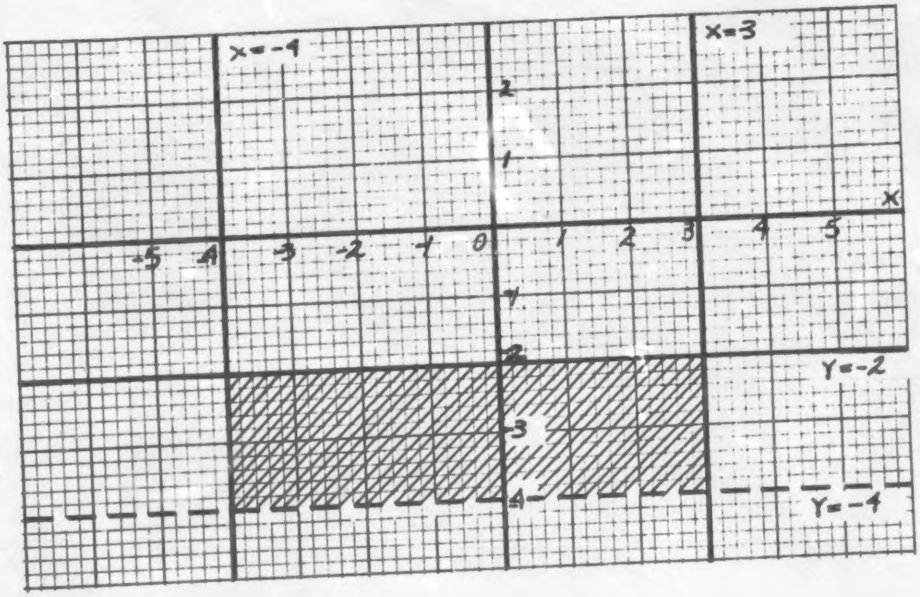
c. $n + b \geq 7$ and $n + b \leq 15$

d. $T > 0$

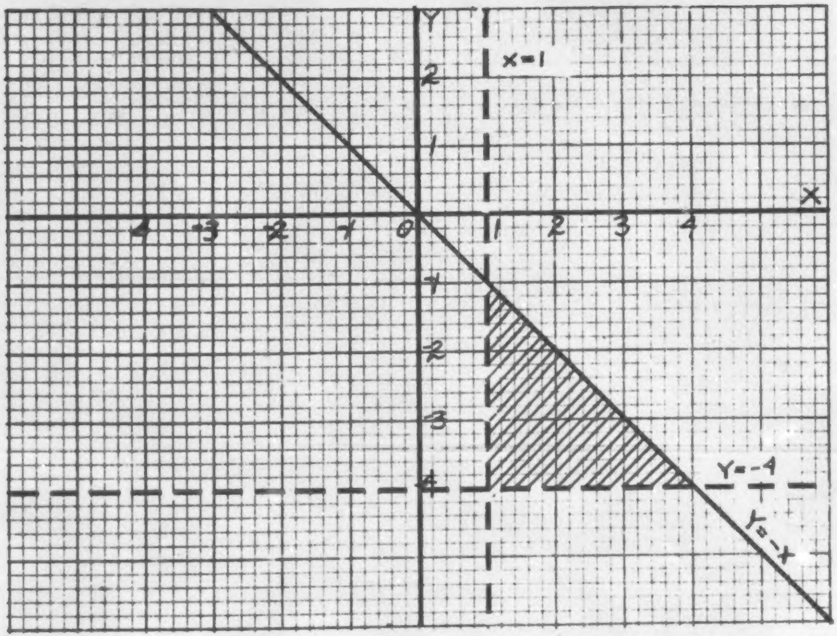
38.



39



40.



41. a. $y \leq 2x + 5$ c. $y \geq -x$

b. $y \leq -\frac{x}{2} + \frac{15}{2}$ d. $y \leq -2x - 3$

42. $a[2, -1, 3] + b[0, -2, 1] \geq [-1, 0, 1]$

$2a \geq -1$

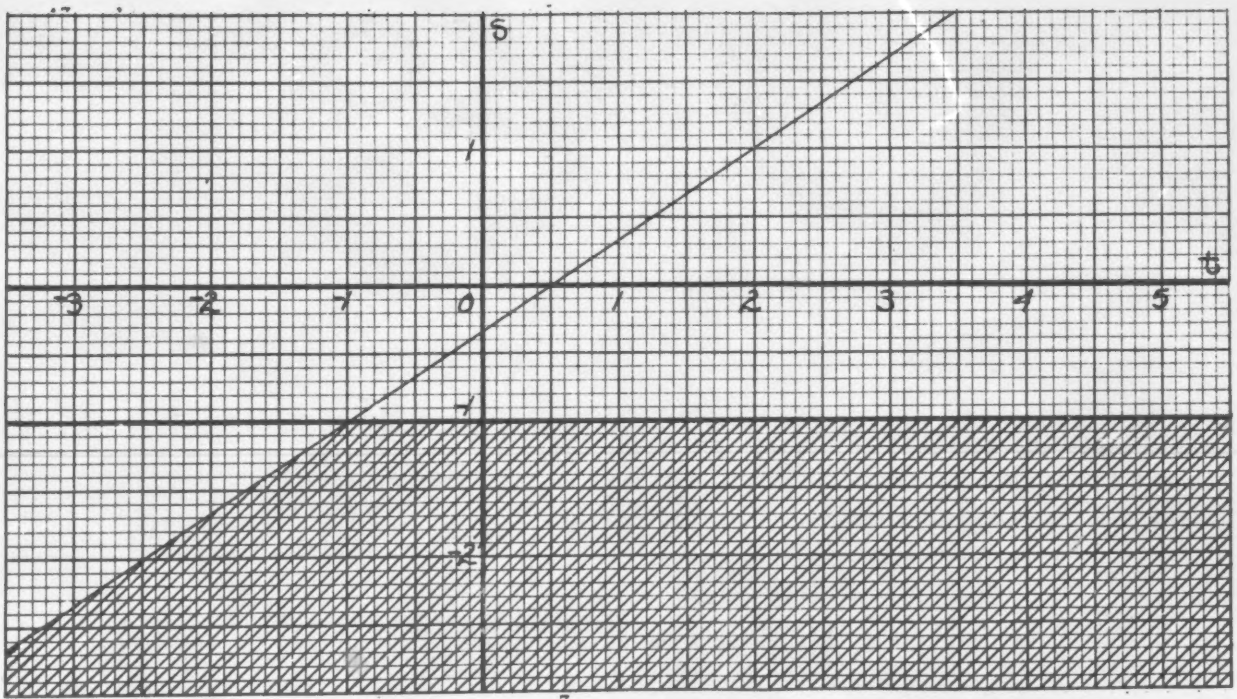
$-a - 2b \geq 0$

$3a + b \geq 1$

43. a. $s \leq -1$

$3s - 2t \leq -1$

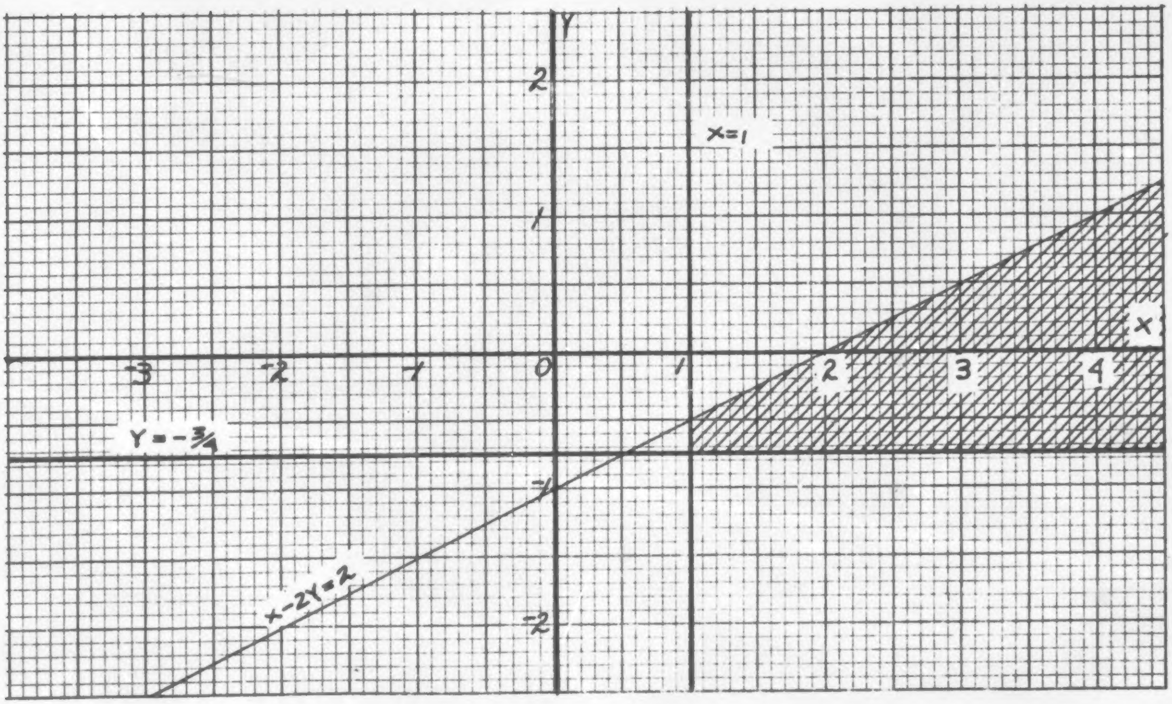
b. See graph on following page.



44. $4y \geq -3$ → $y \geq -\frac{3}{4}$

$x - 2y \geq 2$ → $y \leq \frac{1}{2}x - 1$

$x \geq 1$ → $x \geq 1$



71

45. Let x = number of 100 g portions of lentils

y = number of 100 g portions of soybeans

then

$$x[25, 60] + y[35, 35] \geq [210, 455]$$

This leads to the following two inequalities

$$y \geq -\frac{5}{7}x + 6 \qquad y \geq -\frac{12}{7}x + 13$$

Also we have $x \geq 0$ and $y \geq 0$.

The region of feasible solutions is shown on the graph (see following page.)

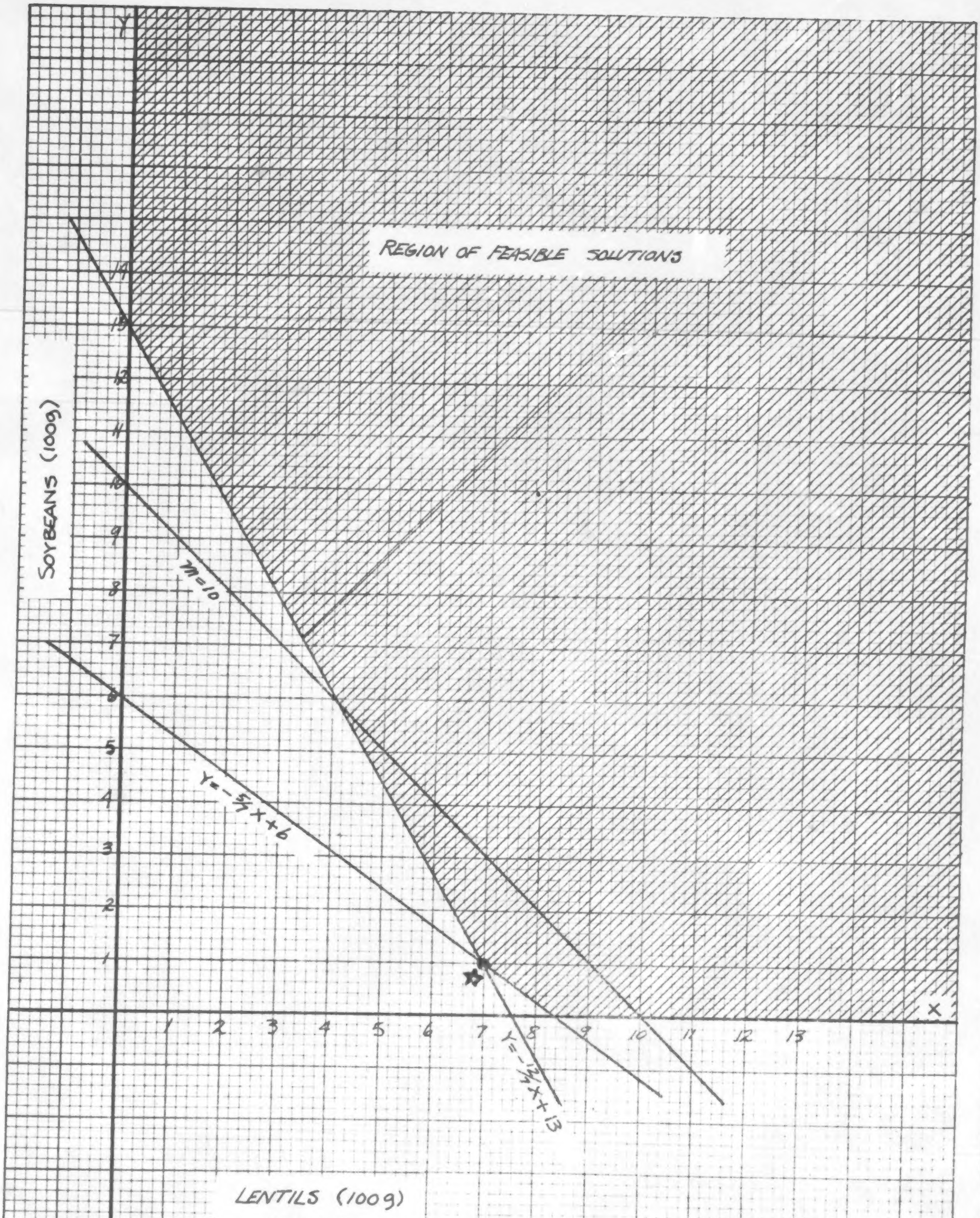
The mass of any combination (in 100 g units) is given by the formula

$$m = x + y$$

A line for $m = 10$ is included on the graph.

The minimum m will occur at the starred point. The coordinates are (7, 1).

We therefore conclude that the combination of least mass consists of 700 g of lentils and 100 g of soybeans with a total mass of 800 g.


REGION OF FEASIBLE SOLUTIONS
SOYBEANS (100g)
LENTILS (100g)
Y
X
14
13
12
11
10
9
8
7
6
5
4
3
2
1
1
2
3
4
5
6
7
8
9
10
11
12
13
m=10
Y=-5/7 X+6
Y=-13/7 X+13

LESSON 23: INTRODUCTION TO STATISTICS

OBJECTIVE:

The student will perform a coin-dumping experiment, will tally the results and will compute the mean number of heads per trial.

PERIODS RECOMMENDED:

Two

SUPPLIES:

16 coins per team of two students with some extra

OVERVIEW AND REMARKS:

The most efficient method of collecting coin-dumping data is for students to work in teams of two, one dumping and one recording. Each team's tally sheet should resemble the one shown in Section 23-3. You should use a similar design to tally team totals on the chalkboard.

Since we will eventually be calculating relative frequencies from the formula,

$$R = \frac{O}{N}$$

R = relative frequency

O = observed frequency

N = total number of trials

it simplifies calculations greatly if N is a round number. Therefore, the number of trials assigned to each team should be chosen so that the total number of trials for the whole class is convenient. If the total number of trials ends up to be a number like 487 instead of 500, you can save yourself and your students many headaches later by taking 16 coins out of your pocket and recording the results of 13 (or the appropriate number) more trials.

You will be responsible for calculating the mean number of heads per trial for the total class data. Each team is responsible for the calculation of the mean number of heads per trial for their own data. The total class mean will generally be closer to eight than most of the teams' individual means.

Be sure to save the total class data. The information will be needed in the following lessons. As new techniques are developed they will be applied to these data.

74

KEY--PROBLEM SET 23:

1. F

2. Perhaps almost everyone in the general population smokes. Other answers are also possible.

3. a. We must show that a higher percentage of smokers than nonsmokers develop emphysema.

   b. We must have data on a large number of people so we can be confident of our results.

4. c, d

5. Individual answers.

6. a, b, c, f

7. Individual answers

8. Individual answers

9. f

10. a. 0 and 16 heads

    b. 8 heads

    c. less

11. a. 0h and 6h

    b. 3h

    c. 3h

    d. [illegible]h

12.

Most Often	5h	6h	7h	8h	9h	10h	Least Often
		4h	3h	2h	1h	0h	

13. a. 11

    b. n + 1

14. 50

15. Zero

16. 3 heads

17. 62 heads

18. No

19. The observed pattern is the opposite of the expected pattern, i.e., the farther away from 2 heads the more often the event was observed.

20. When the number of trials is small it is more common to see unexpected patterns than it is when the number of trials is large.

21. 5

22. 20

23. 2

24. 2

25. 0

26. 40

27. 2

28. a. 7

b. 102

29. 7

30. a. 130-149 mm Hg

b. 210-229 mm Hg

31. 135-139 mm

32. 120-124 mm

33. 115-119 mm

34. 68

35.

Event	Observed Frequency
0	6
1	27
2	80
3	33
4	7

36.

Event (Height in cm)	Observed Frequency
150-154	7
155-159	17
160-164	41
165-169	36
170-174	15
175-179	9
180-184	2

LESSON 24: TALLY HO!

OBJECTIVES:

The student will calculate the relative frequency of an event, given the observed frequency and the total number of trials.

The student will calculate the expected frequency of an event, given the probability and the total number of trials.

PERIODS RECOMMENDED:

One

SUPPLIES:

Total class data from the coin-dumping activity of Lesson 23.

OVERVIEW AND REMARKS:

Relative frequencies for the total class coin-dumping data are calculated in Problem 27. You should make sure that the data are available in some form for the entire class, i.e., dittoed, copies on the chalkboard, etc.

KEY--PROBLEM SET 24:

1. frequency distribution

2. F

3. Never to always

4. a. relative frequency
   b. observed frequency
   c. total number of trials
   d. R

5. happened or occurred

6. predictive

7. probability, relative frequency

8. a. probability
   b. expected frequency
   c. total number of trials
   d. E

9. more often than any other event

10. a. $\frac{1}{6}$
   b. 90
   c. 15
   d. no
   e. yes

11. the number of trials

12. 62

13. $R = \frac{O}{N}$

$= \frac{2}{20}$

$= .10$

14. $R = \frac{O}{N}$

$= \frac{7}{20}$

$= .35$

15. zero heads

16. b

17. 8

18. 54

19. greater than 12.5 to 13.0

20. 3.7

21. $R = \frac{O}{N}$

$.204 = \frac{O}{54}$

$(54)\ (.204) = 11.016$

$\simeq 11$

22. $.037 = \frac{O}{54}$

$(54)\ (.037) = 1.998$

$\simeq 2$

23. 1.000 or 1

24. $p = \frac{E}{N}$

$.5 = \frac{E}{20}$

$10 = E$

25. 60

26. a. .468
   b. .88
   c. .10
   d. .372

27. Answers depend upon class data

LESSON 25: A DESCRIPTIVE-FREQUENCY HISTOGRAM

OBJECTIVE:

The student will construct and interpret frequency histograms and relative-frequency histograms.

PERIODS RECOMMENDED:

One

SUPPLIES:

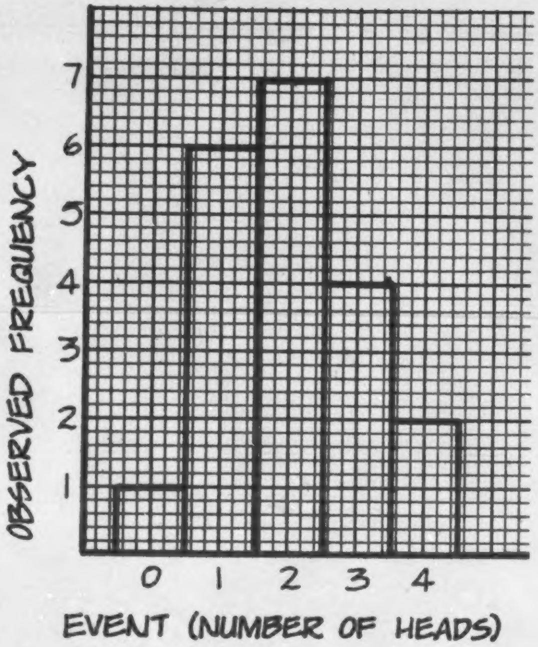
Total class data from the coin-dumping activity of Lesson 23

OVERVIEW AND REMARKS:

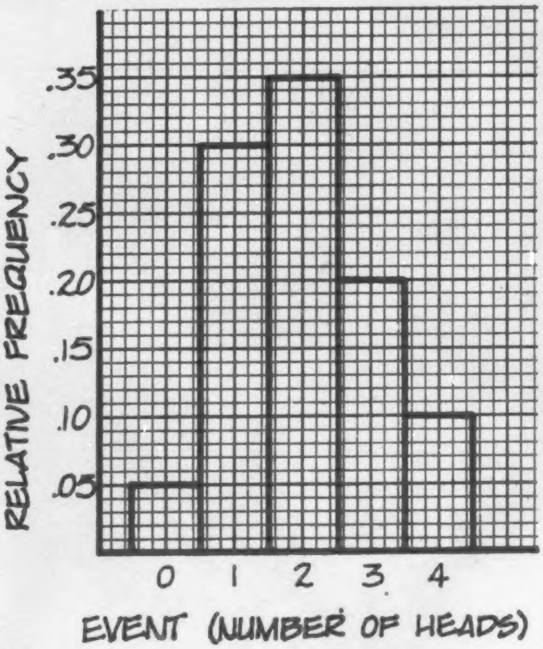
A relative-frequency histogram for the coin-dumping data is called for in Problem 19.

KEY--PROBLEM SET 25:

1.



2.



3. .15

4. .65

5. .65

6. one head or three heads

7. .32

8. .38

9. .42

10. 1 event away

11. .166

12. .259

13. .519

14. .186

15. .592

16. 1.000

17.

Event #	Observed Frequency
1	2
2	3
3	9
4	12
5	11
6	12
7	2
8	3

18. .648

19. Answers will depend upon total class data.

LESSON 26: YET ANOTHER FREQUENCY HISTOGRAM

OBJECTIVE:

The student will use the concept of combined relative frequency to construct and interpret mean-centered histograms.

PERIODS RECOMMENDED:

One

SUPPLIES:

Total class data from the coin-dumping activity of Lesson 23

OVERVIEW AND REMARKS:

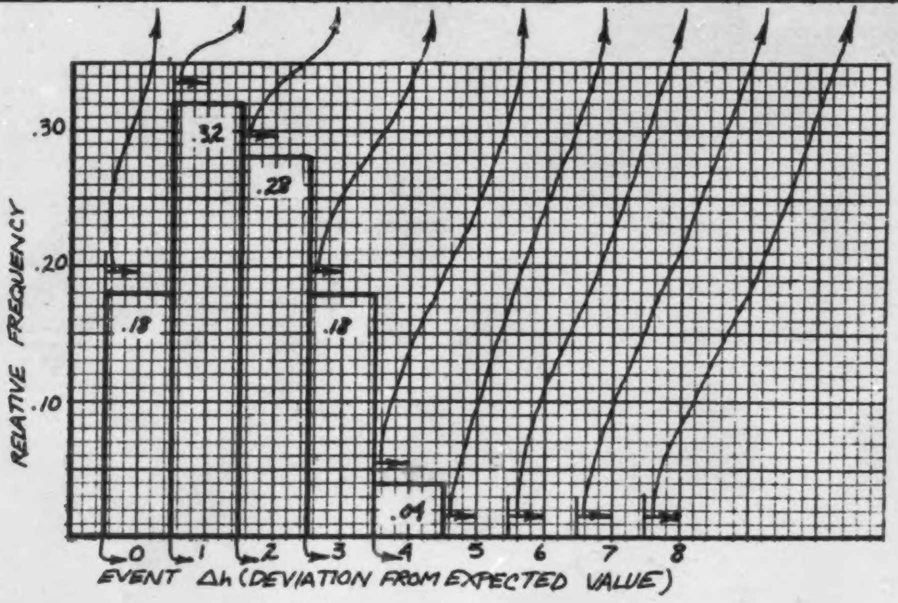
A mean-centered relative frequency histogram is called for in Problem 19.

KEY--PROBLEM SET 26:

1. 7, 9
2. 1
3. .32
4. 2
5. .28
6. b
7. 2
8. hopefully
9. 1
10.

TABLE OF COMBINED EVENTS

EVENT (Deviation from expected value) Δh	0 or more	1 or more	2 or more	3 or more	4 or more	5 or more	6 or more	7 or more	8 or more
RELATIVE FREQUENCY	1.00	.82	.50	.22	.04	0	0	0	0

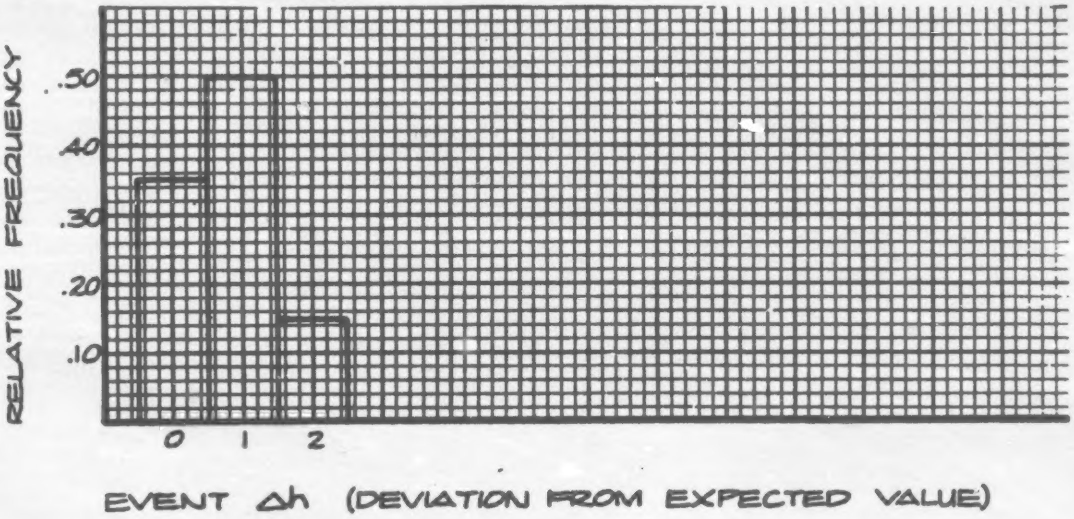


11. .50

12.

EVENT (Deviation from Δh expected value)	0	1	2
RELATIVE FREQUENCY	.35	.50	.15

13.



14.

EVENT (Deviation from Δh expected value)	0 or more	1 or more	2 or more
RELATIVE FREQUENCY	1.00	.65	.15

15. 1

16. a. 7
    b. 12, 2
    c. 2
    d. 2
    e. 0, 1, 2, 12, 13, 14
    f. 0, 1, 2, 3, 4, 10, 11, 12, 13, 14
    g. 4, 5, 6, 7, 8, 9, 10

83

17. a. 6

b. 6

c. 5, 7

d. 5, 6, 7

e. 1, 11

f. 0, 1, 2, 3, 4, 8, 9, 10, 11, 12

g. 4, 5, 6, 7, 8

18. For $\Delta h \geq 0$, $O = 50$

For $\Delta h \geq 1$, $O = 41$

For $\Delta h \geq 2$, $O = 25$

For $\Delta h \geq 3$, $O = 11$

For $\Delta h \geq 4$, $O = 2$

For all others $O = 0$

19. Answers will depend upon total class coin dumping data.

LESSON 27: FREQUENCY DISTRIBUTIONS FROM A THEORETICAL POINT OF VIEW

OBJECTIVE:

The student will use theoretical frequency distributions to calculate the expected frequencies of events.

PERIODS RECOMMENDED:

One

SUPPLIES:

Total class data from the coin-dumping activity of Lesson 23.

OVERVIEW AND REMARKS:

The coin-dumping data are required in Problems 3b and 22.

KEY--PROBLEM SET 27:

1. Any two of the following

   a. Tossing coins and tabulating takes time.

   b. Coins might be lopsided.

   c. The method of dumping coins might not be truly random.

2. Probabilities are expected relative frequencies, while relative frequencies are computed from data already collected.

3. a.

EVENT (number of heads)	p	R	|p - R|
5	.07	.10	.03
6	.12	.16	.04
7	.17	.16	.01
8	.20	.18	.02
9	.17	.16	.01
10	.12	.12	0
11	.07	.08	.01
12	.03	.04	.01

   b. Answer depends upon total class coin-dumping data.

4. 8

5. 3

6. c

7. ~175

8. ~24

9. About 66,667 trials

10. 5 trials

11. a. 4000 trials

    b. 4000 trials

12. a. It is symetrical to the left and right around 8 heads.

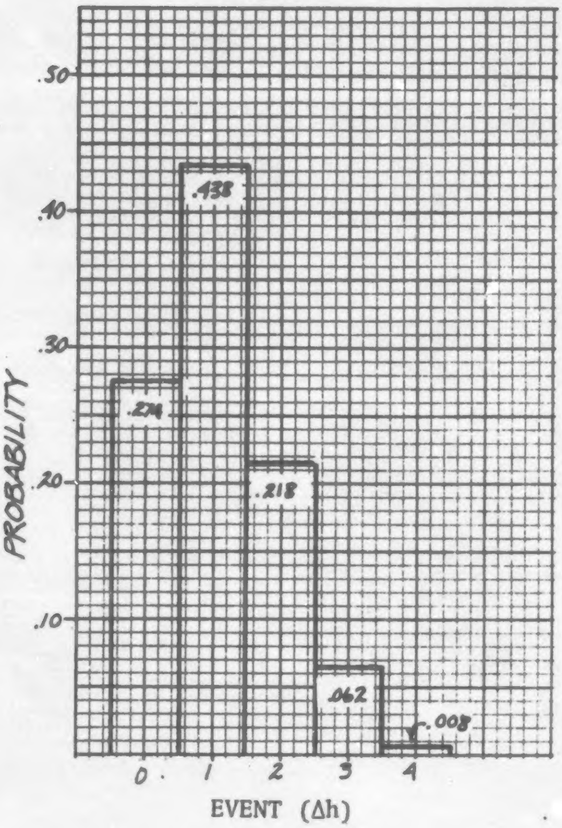
    b. It has a shape similar to a bell.

13. 6

14. 100,000

15. 8

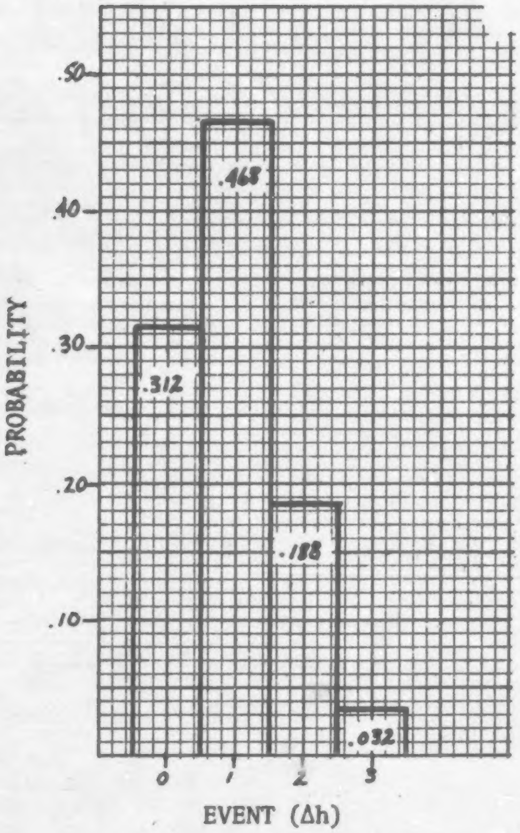
16.



17.

	EVENT	0 or more	1 or more	2 or more	3 or more	4 or more
a.	COMBINED PROBABILITY	1.000	.726	.288	.070	.008
b.	For N = 50, E =	50	36.3	14.4	3.5	.4

18.



19.

	EVENT	0 or more	1 or more	2 or more	3 or more
a.	COMBINED PROBABILITY	1.000	.688	.220	.032
b.	For N = 50, E =	50	34.4	11	1.6

20. The sum of all the relative frequencies, either observed or theoretical, must be 1.

21. The probabilities are so low in these categories that there were not enough trials for the events to be observed.

22. Answer depends upon total class coin-dumping data.

LESSON 28: THE CHI-SQUARE PROCEDURE

OBJECTIVE:

The student will use the chi-square procedure as a short-cut to the combined events calculations of Lessons 26 and 27.

PERIODS RECOMMENDED:

One or two

SUPPLIES:

One coin per student

One pair of dice per two students (optional)

OVERVIEW AND REMARKS:

You may wish to generate more χ^2 tables of the type described in Section 28-1. An endless number of drill problems may be generated in this manner. For example, you could enlist the cooperation of 16 students, each flipping a single coin, to generate 3 or 4 more χ^2 contingency tables. For the 16-coin tables, p may be found by two methods.

1. By referring to the table of combined probabilities in Section 27.
2. By performing χ^2 calculations.

Another possibility is to use a larger or smaller number of coins than 16. Then p may be determined only by the χ^2 method. You might use an odd number of coins at least once to demonstrate that expected frequencies need not be whole numbers.

Still another possibility is to use dice, each student rolling a single die, to produce tables like

SIX	$4^{(5)}$
NOT SIX	$26^{(25)}$
TOTAL	30

OR

5 OR 6	$15^{(10)}$
NOT 5 OR 6	$15^{(20)}$
TOTAL	30

PREPARATION FOR FUTURE LESSONS:

The next lesson requires information on the sex composition of your schoool, to be obtained from school records. Specifically, three numbers are needed, the total number of students, the number of males and the number of females. You may wish to appoint a committee during this class period to gather the information for the next lesson.

In addition to sex, your school office may have other information that would be interesting to analyze, for example, ethnic background. The only restriction is that the information divide the class into only two categories such as black-white, over 5' 6"-under 5' 6", etc. One of these alternatives will be necessary if your school happens to be all male or all female.

KEY--PROBLEM SET 28:

1. expected

2. observed

3. $$\frac{(|0 - E| - \frac{1}{2}^2)}{E} = \frac{(|11 - 8| - \frac{1}{2})^2}{8}$$

$$= \frac{(2.5)^2}{8}$$

$$\approx 0.8$$

4. 0.8

5. 1.6

6. .20

7. 3

8. .21

9. smaller

10. 99

11. .46

12.

HEADS	$3^{(8)}$
TAILS	$13^{(8)}$

13. 5.0 or 5.1, depending on when rounding is done.

14. .03

15. 3

16.

HEADS	$57^{(50)}$
TAILS	$43^{(50)}$

17. 1.7

18. .18

19. no

LESSON 29: CHI SQUARE AND YOUR BIOMED CLASS

OBJECTIVES:

The student will compute expected frequencies and construct two-cell contingency tables.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 29:

1. .45
2. .55
3. 9
4. 11
5.

MALES	$7^{(9)}$
FEMALES	$13^{(11)}$
TOTAL	20

6. 4
7. 495
8. 6.12
9. 4
10. 126
11. 5.76, no
12. 39
13. 6.9
14. West African
15. tongue rollers: 335

    non-rollers: 165

16.

ROLLERS	$320^{(335)}$
NON-ROLLERS	$180^{(165)}$
TOTAL	500

17. 2
18. 400,000
19. 9
20.

HITCH HIKERS THUMB	$12^{(9)}$
STRAIGHT THUMB	$13^{(16)}$
TOTAL	25

21. 27.2

LESSON 30: HOW ODD (STATISTICALLY) IS YOUR CLASS?

OBJECTIVE:

The student will apply the chi-square procedure to two-cell contingency tables.

PERIODS RECOMMENDED:

One

KEY--PROBLEM SET 30:

1. .45

2. .50

3. 50

4.

ROLLERS	$56^{(67)}$
NON-ROLLERS	$44^{(33)}$
TOTAL	100

5. 5.0

6. .03

7. Not randomly selected. When $p < .05$ the assumption of random selection is rejected.

8. 5 times out of 100

LESSON 31: REVIEW

OBJECTIVE:

The student will solve problems related to the objectives for Lessons 23 through 30.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

Some or all of the problems may be assigned. However, certain groups of problems are best assigned as a whole.

UNBREAKABLE GROUPS	TOPIC REVIEWED
1-11	Relative Frequency
12-29	Table of Combined Relative Frequency
30-38	Table of Combined Probability
39-46	χ^2 Procedure

If you wish to assign only some of the problems in a group, make sure that they follow a logical sequence.

KEY--PROBLEM SET 31:

1. 11

2. 31

3. 6.35-6.39

4. 6.10-6.14

5. the number of trials (or total)

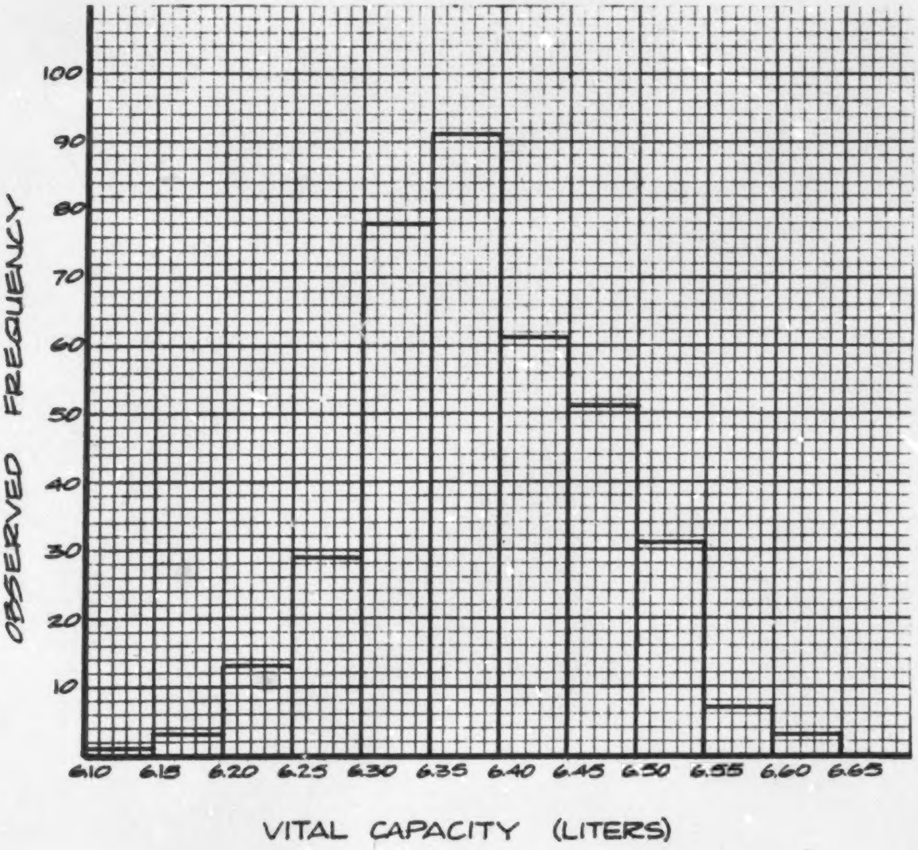
6. 6.35-6.39 liters

7. 24.7

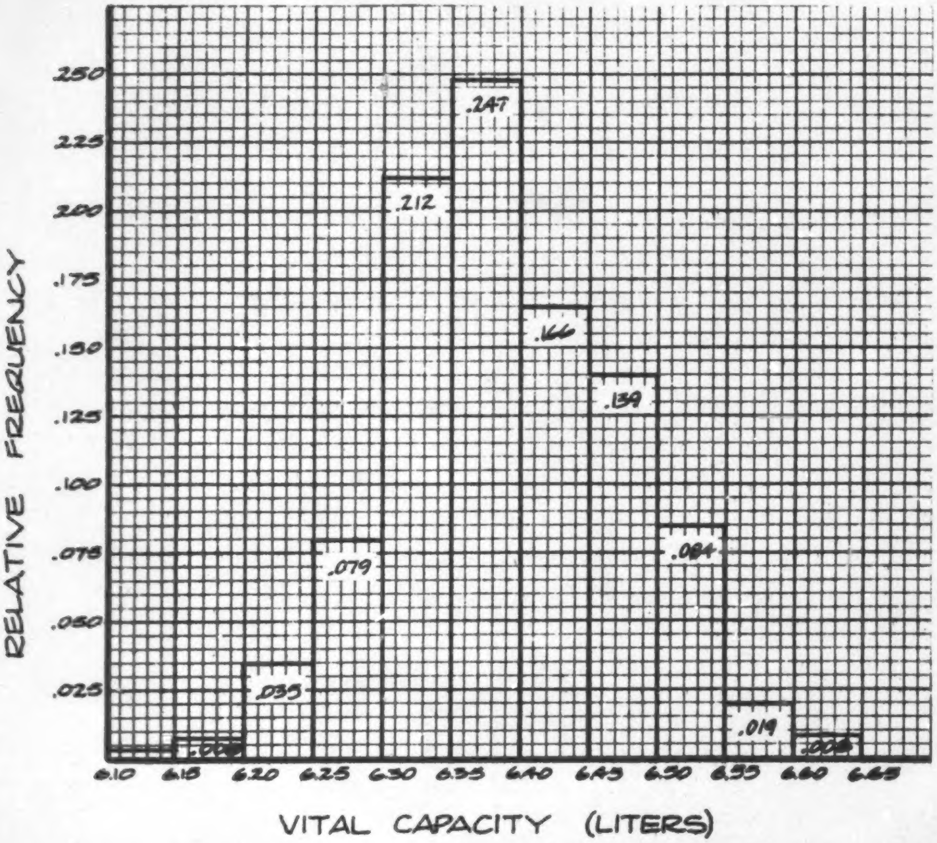
8. 6.50-6.54 liters

9. vital capacity in the range 6.15-6.19

10.

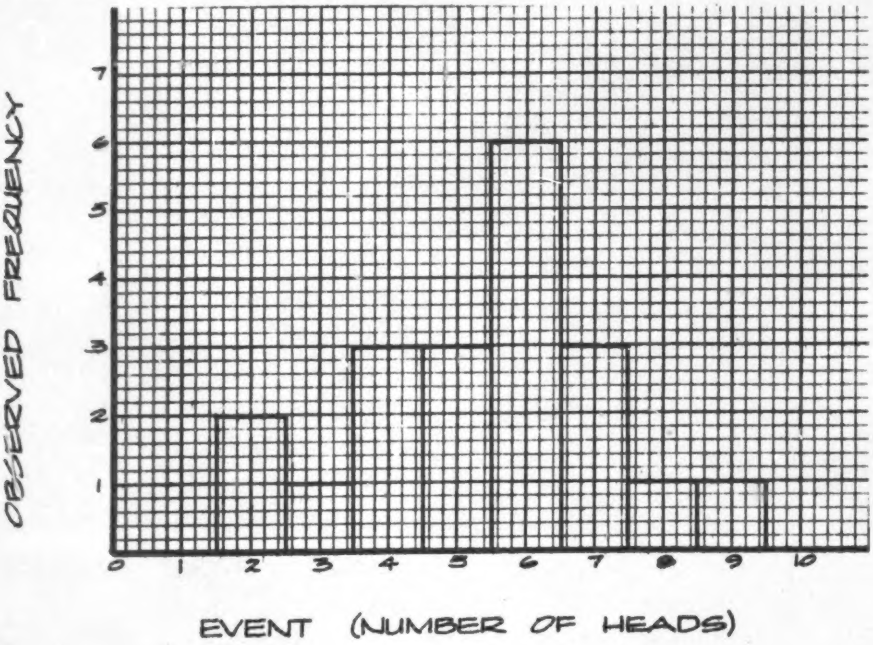


11. a.



b. Same shape as in Problem 10.

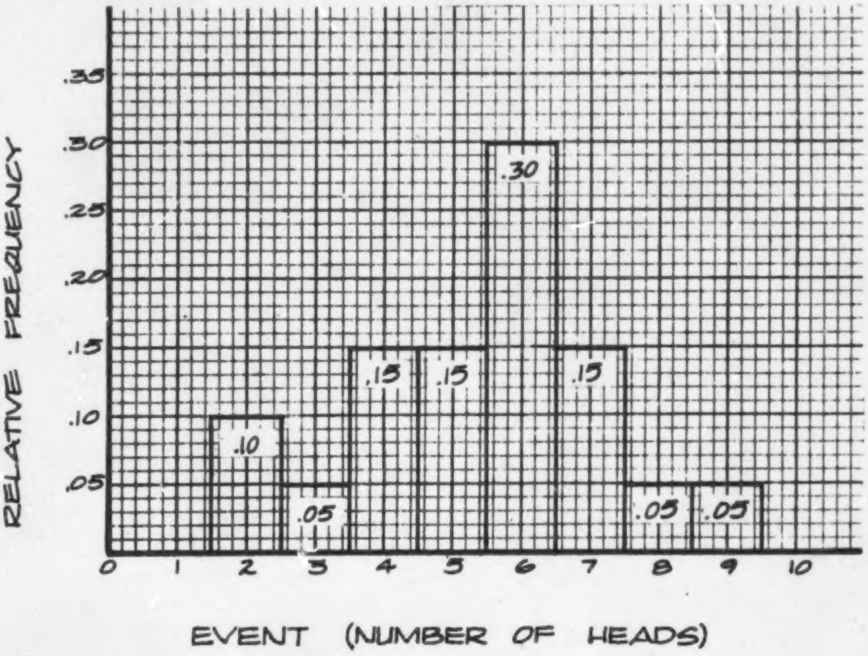
12.



13.

EVENT (number of heads)	0	1	2	3	4	5	6	7	8	9	10
RELATIVE FREQUENCY	0	0	.10	.05	.15	.15	.30	.15	.05	.05	0

14.



15. .15

16. .45

17. .85

18. 2, 6

19. 1.00

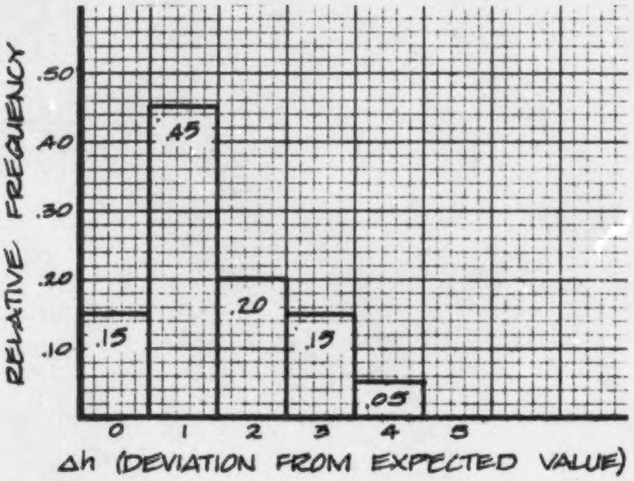
20. 5

21. .45

22.

EVENT Δh (Deviation from expected value)	0	1	2	3	4	5
RELATIVE FREQUENCY	.15	.45	.20	.15	.05	0

23.



24.

EVENT (Deviation from expected value)	0 or more	1 or more	2 or more	3 or more	4 or more	5 or more
RELATIVE FREQUENCY	1.00	.85	.40	.20	.05	0

25. 2

26. 5

27. 8

28. 44

29.

EVENT Δh (deviation from expected value)	0	1	2	3	4	5
PREDICTED RELATIVE FREQUENCY, OR PROBABILITY	.246	.410	.234	.088	.020	.002

30. 7

31. 10,000

32.

EVENT Δh (deviation from expected value)	0 or more	1 or more	2 or more	3 or more	4 or more	5 or more
COMBINED PROBABILITY	1.00	.754	.344	.110	.022	.002

33. .110

34. 11.0%

35. expected

36. observed

37. a. 1.25

b. 1.25

38. 2.5

39. .12

40. 3

41. .110

42. a. 200 c. 100

b. 100 d. 108, 92

e.

HEADS	$108^{(100)}$
TAILS	$92^{(100)}$
TOTAL	200

f. $\chi^2 \approx 1.1$

g. .30

LESSON 32: DRUGS AND THE PLACEBO EFFECT

OBJECTIVE:

The student will compute expected frequencies and chi-square for a four-cell contingency table.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

The 2 × 2 table is introduced for the first time in a context that is conceptually simpler than the Cola Activity which follows in the next lesson.

PREPARATION FOR FUTURE LESSONS:

Lesson 33 requires advance preparation. Be sure to examine it early enough to obtain the materials required.

KEY--PROBLEM SET 32:

1. There is no difference between Dramamine and the placebo in terms of preventing vomiting.

2. 91

3. 125

4. did

5. 45.5

6. did not

7. 62.5

8.

	DRAMAMINE	PLACEBO	TOTAL
SICK	$31^{(45.5)}$	$60^{(45.5)}$	91
NOT SICK	$77^{(62.5}$	$48^{(62.5)}$	125
TOTAL	108	108	216

9. $\chi^2 \simeq 14.89$ (calculator answer)

$\chi^2 \simeq 14.37$ (result if denominators were rounded)

10. .01

LESSON 33: THE COLA ACTIVITY

OBJECTIVES:

The students will conduct an experiment to gain data on their brand and taste preferences for two brands of cola. They will then analyze the data by the chi-square method, to determine the relation, if any, between these preferences.

SUPPLIES:

Materials for the Cola Experiment (enough for 40 subjects)

- 1 quart Pepsi-Cola
- 1 quart Coca-Cola
- 2 identical quart jars or 1-liter Erlenmeyer flasks
- 1 ice chest with ice or refrigerator
- 80 small paper cups
- 1 coin per student

PERIODS RECOMMENDED:

Two or three

READING SEQUENCE:

Student Text-Instructor's Guide

DIRECTIONS FOR THE COLA ACTIVITY:

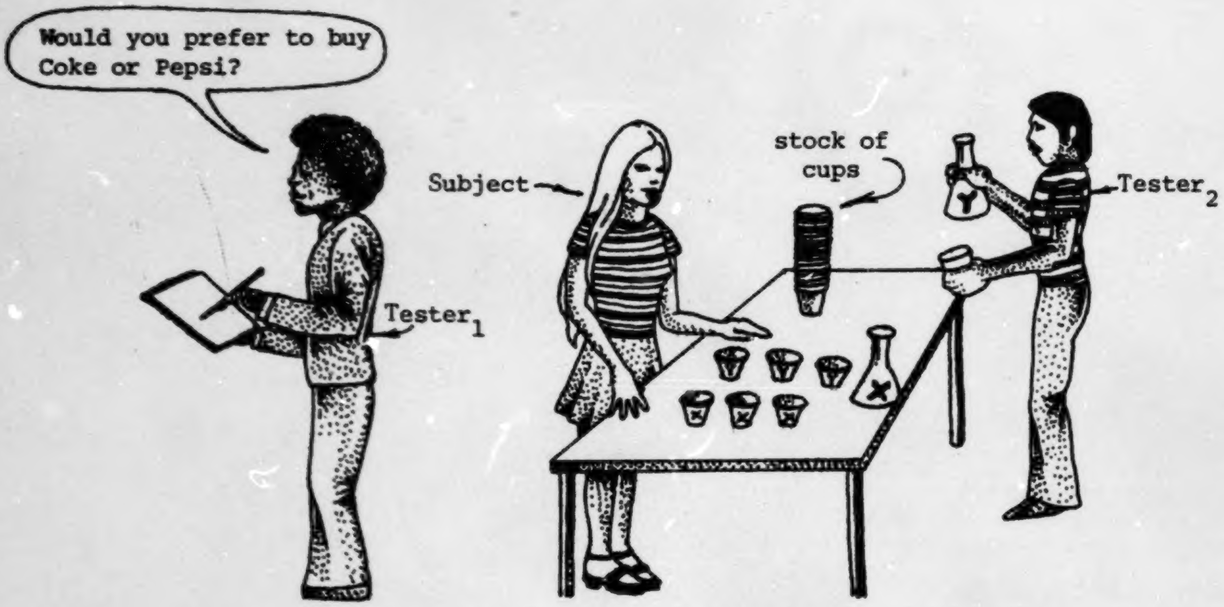
The basic idea of this activity is the analysis of responses to two questions. The two questions are:

Question 1: "Do you prefer to buy Coke or Pepsi?" in the context of which one they would pick up in the supermarket.

Question 2: "Do you prefer the taste of x or y?" where x and y are Pepsi and Coke in unidentified Erlenmeyer flasks.

You will survey the class and demonstrate how to analyze the results. Then, if you wish, you can mention that a larger sample would produce more convincing results. Turn them loose to survey as many other students as they can.

On the following page is a diagram of a convenient testing set-up.



A. Tester$_1$ records the name and brand preference.

B. Tester$_2$ marks the cups and pours x and y into them. When Tester$_1$ has recorded the brand preference, Tester$_2$ gives the subject the samples of x and y to take to his seat to taste. He should be cautioned (1) not to talk to neighbors and (2) to record which taste he prefers. It's okay to tell them that x and y are Coke and Pepsi, not necessarily respectively.

When the dispensing is finished, record the taste preferences. The data sheet should have three columns as shown below.

NAME	BRAND	TASTE
Elmo Elbo	*C*	*y*
Myo Pia	*P*	*y*
Theo Remin	*C*	*x*
I. Ball	*P*	*y*

The results can be tallied quickly be entering the results in a table like the one on the following page. Elmo's results from the preceding table are tallied on the following page.

		BRAND PREFERENCE		
		COKE	PEPSI	TOTAL
TASTE PREFERENCE	COKE (X)			
	PEPSI (Y)	|		
	TOTAL			

The datum for each student is tallied in a smiliar fashion.

GROUND RULES

1. Both bottles of Coke and Pepsi should be icy cold. Warm colas are readily identifiable. Also, if they are at different temperatures, people tend to prefer the colder cola.

2. For the "in-class" survey, have any students who are helping to administer the test wait until last to take the test themselves. Otherwise, their personal preferences may somehow influence the data they collect. Ideally, the testers should not know the identities of the x and y brands. It is a well-known fact that those who are administrating the test can influence the results.

3. The colas should be pre-poured as indicated in the diagram of the testing set up. (The foams are different colors.)

4. Emphasize that there are no "wrong" answers. Subjects should not be trying to make their choice of x or y coincide with their brand preference. They should be trying to decide which taste they prefer.

5. Students should not be allowed to discuss or comment on their impressions. Some may be able to detect flavor differences better than others and could influence or prejudice the results.

OVERVIEW AND REMARKS:

Before reading this section you should read the Student Text.

The total activity occurs in three parts. First Biomed students are used as subjects. χ^2 and p are determined for the table produced.

Second, you lead the class in the generation of random χ^2 tables. χ^2 and p are calculated for each of these tables. These tables will provide insight into what χ^2 and p measure in this context. Once the students have mastered the concepts and calculations associated with the analysis, then Biomed students conduct the activity using other students as subjects. There are two objectives in this. First, it gives Biomed students an opportunity to see whether other groups show patterns similar to their own. Second, experience has shown that it is great for morale. Biomed students are seen as bearers of gifts of Coke and Pepsi. This does not harm the image of Biomed students.

You will probably experience much less difficulty with the mathematical aspects of the analysis than with the interpretation of the results. The second part of the activity is designed to assist your efforts in this direction. It will provide concrete, randomly produced tables for comparison with the table containing the actual data from the Cola Activity. It follows the line of reasoning described in Section 33-3, "The Coin-Flippers."

1. Require each student to record his or her own taste choice. If anyone has forgotten, the information may be recovered from the data sheet for the Cola Activity.

2. Draw a contingency table on the chalkboard. Fill in only the TASTE PREFERENCE TOTALS for your own class data. We use the numbers from Elmo's Pudworthy class for purposes of illustration only. Your numbers will likely be different.

TASTE PREFERENCE	COKE			35
	PEPSI			15
				50

Whatever the filled-in numbers are, they will not change. They will be the same for each table to be generated.

3. Have each student flip a coin. Then, call for a show of hands for each box in the contingency table. For example, ask everybody who chose Coke taste and had heads on the coin flip to raise their hands. All other observed frequencies may be derived from this number, or a show of hands may be asked for each box. Below we have the results of one such trial for Elmo's class.

		HEADS	TAILS	
TASTE PREFERENCE	COKE	$20^{(17.5)}$	$15^{(17.5)}$	35
	PEPSI	$5^{(7.5)}$	$10^{(7.5)}$	15
	TOTAL	25	25	50

4. Generate as many of these randomly produced contingency tables as desired. Then, a probability determination for a single table may be assigned to a small group of students. Some of the probabilities may be as low as the probability for the actual Cola-Pepsi contingency table, reinforcing the point made in Section 33-3. One more detail. Conventionally, if a single expected frequency ≤ 5, then a χ^2 analysis will not give very accurate probabilities. If this has occurred in the first part, then the results for the Biomed class may be pooled with the results gathered in the last part of the activity.

Finally, we realize that our arguments about the implications of the results are not airtight. For example, it could be argued that people don't ordinarily drink cola icy cold. Since the tastes are reportedly more distinguishable as the temperature rises, it is possible that taste is a real factor in brand choice at normal drinking temperature. In this case, random results imply only that the tastes are indistinguishable when the colas are icy cold.

At any rate you should encourage your students to try to find flaws in the experimental design and then ask them to design another activity in which the flaws have been corrected. For example, the temperature of soft drink dispensers could be found. Then the activity could be rerun with the colas served at that temperature.

Finally, if you get random results ($p \geq .05$), then you may want to try yet another activity. For example, the activity could be run and the same cola drink poured from both Coke and Pepsi bottles. Do not tell the test subjects that they are drinking the same cola. Since everybody would be tasting the same drink, stated taste preferences could not possibly be based on taste or any other sense-related feature of the drink. If $p < .05$ for this activity, then it can be assumed that stated taste preference is associated with brand recognition, i.e., recognition of the bottles.

KEY--PROBLEM SET 33:

1. 30

2. 35

3. 11

4. 9

5. a, b

		BRAND PREFERENCE		
		COKE	PEPSI	TOTAL
TASTE PREFERENCE	COKE	$26^{(21)}$	$4^{(9)}$	30
	PEPSI	$9^{(14)}$	$11^{(6)}$	20
	TOTAL	35	15	50

c. $\chi^2 \approx 8.1$ (based on approximation given)

d. $p < .01$

e. The pattern in the table deviates significantly from common random patterns. Some sort of cause-and-effect explanation is suggested.

6. a. $\chi^2 \approx 3.645$ (based on approximation given)

b. $p > .05$

c. No kind of cause-and-effect relation can be assumed.

7. a. Table B

b. The sum of the positive responses is greater than the sum of the negative responses (35 > 25).

8. Advertising, attitudes of friends, the taste of the two colas at a different temperature.

9. Taste

LESSON 34: CHI SQUARE PROBLEM SET

OBJECTIVE:

Given the application of the chi-square procedure to a specific problem, the student will interpret the meaning of the resulting probability.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

In this problem set the chi-square procedure is applied to problems from three disciplines, medicine, genetics and social science. The main focus is on the interpretation of χ^2 and p in different contexts.

KEY--PROBLEM SET 34:

1. 38

2. 27

3. 31

4. 14

5. The assumption is that both subgroups represented by the columns are random samples with respect to getting sick. In other words, there is no relationship between eating the food and getting sick.

6. a. $p < .01$

b. Had the subgroups represented by the columns been chosen by a random procedure, the deviations would be as large or larger in 14% of the trials.

7. egg salad

8. macaroni and cheese

9. a. They most likely also ate egg salad.

b. Perhaps the people had actually eaten egg salad but had forgotten about it. Or, perhaps, they hadn't eaten egg salad, but experienced nausea for some other reason.

10. a. 88.5 for each color

b.

Red-eyed females	$86^{(88.5)}$
White-eyed females	$91^{(88.5)}$
	177

11. $\chi^2 \approx .09$

12. $p \approx .76$

13. no

14. It sheds no suspicion on the original assumption that each eye color is equally likely because deviations as large or larger are common random occurrences.

15. a. $p \approx .04$

b. yes

16. It casts doubt on the original assumption. It would be time to dream up a new one.

17. The assumption is that the control and the experimental group are random subgroups with respect to self-rating. In other words, there is no relation between self-rating and the type of feedback received.

18. a. $p < .01$

b. yes

c. no

d. It casts doubt on it. In other words, it is reasonable to suspect that some kind of cause-and-effect relationship is at work.

e. Positive feedback seems to be associated with an increase in one's self-evaluation.

LESSON X: CORRELATION

OBJECTIVE:

Given a table of data, the student will graph the data, estimate the associated correlation coefficient, and evaluate the implications for a cause-and-effect relationship.

PERIODS RECOMMENDED:

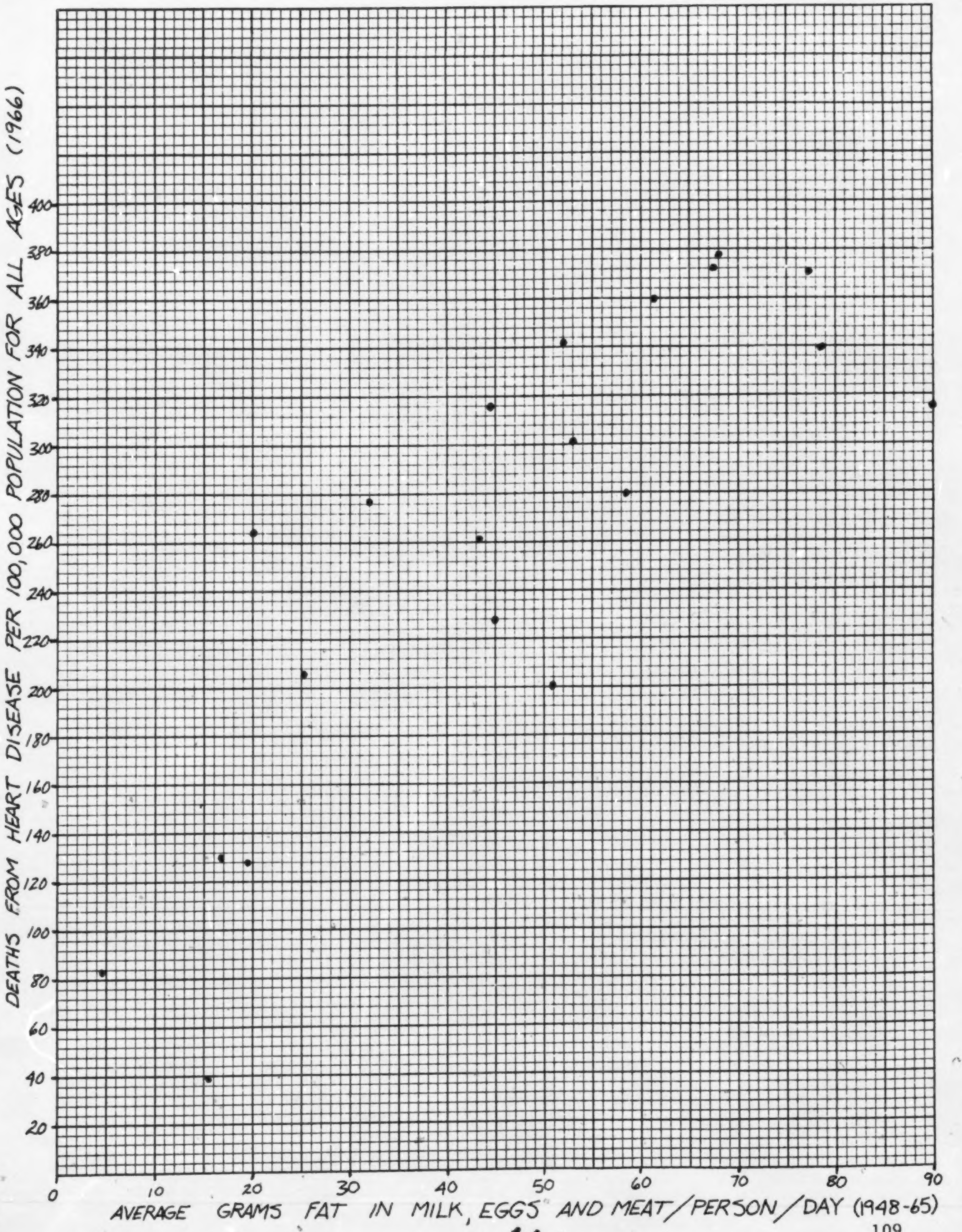
One

OVERVIEW AND REMARKS:

This lesson should be presented as soon after Science Lesson 2 of Unit II as possible. You should read Science Lessons 1 and 2 beforehand. The purposes of this math lesson are twofold. First, it reinforces the discussion of heart-disease-related factors initiated in Science Lesson 2. Second, it presents the idea of correlation, which is important in this course because medical research relies increasingly on statistical studies.

In Unit I, the idea of correlation arose implicitly whenever a set of points was inspected for linearity or when a "best line" was drawn through a set of data points. In the lesson on vital capacity (FEV) it was noted that the cube of height is the best indicator. This can be formulated more exactly by saying that the correlation coefficient for vital capacity vs H^3 is closer to 1 than the correlation coefficients for vital capacity vs wieght, vital capacity vs surface area, etc. It may be helpful to discuss earlier lessons as examples of the ideas presented in this one.

KEY--PROBLEM SET X:



2. yes

3. The true value is .83. The graph in the text based on sugar intake and heart disease is probably most helpful in making an estimate.

4. No. Based on the graph alone, it is possible that heart disease causes increased fat intake or that both heart disease and higher fat intake are caused by some third factor.

LESSON Y: UNCERTAINTIES IN THE VITAMIN C ANALYSIS

OBJECTIVES:

The student will list sources of uncertainty in the vitamin C analysis activity.

The student will list factors which need to be considered in an estimate of the uncertainty associated with each instrument used in the vitamin C analysis activity.

When given a sample numerical value for the absolute uncertainty of a measurement, the student will calculate the relative uncertainty.

PERIODS RECOMMENDED:

One

OVERVIEW AND REMARKS:

This is another "floating" lesson. You and the science instructor will have to coordinate your schedules so that this lesson occurs just before the students perform the vitamin C activity in science (LA-20). This lesson will serve the dual purpose of familiarizing the students with the lab procedure and sensitizing them to sources of uncertainty in the procedure. Be sure that the students understand that they are to return to the next math class with a specific set of numbers, i.e., estimates of the uncertainty of the instruments used in the laboratory activity. In the next lesson (Lesson Z), these numbers will be used to calculate the uncertainty of their final results.

KEY--PROBLEM SET Y:

1. 25 ± 3%
2. 1.8 ± 2.7%
3. 70.2 ± 1.2%
4. 4 ± 4.3%
5. .007 ± .16%
6. .04 ± .04%
7. 20.0 ± .4 ml
8. 30.00 ± .35 ml
9. 28.70 ± .22 ml
10. 14.21 ± .02 g
11. 137.25 ± .07 g
12. 44.700 ± .002 g
13. a. a balance and a volumetric flask
    b. relative, sum, relative
14. a. buret and pipet
    b. buret
    c. buret
15. a. 1%
    b. .5%
    c. 1.5%
16. a. 40 ± .2 ml
    b. .5%
17. a. balance
    b. volumetric flask
    c. pipet
    d. buret
    e. buret
    f. pipet
18. 2.6%

LESSON Z: A SAMPLE ASCORBIC ACID UNCERTAINTY CALCULATION

OBJECTIVE:

The student will calculate the uncertainty of the final results of the vitamin C analysis.

PERIODS RECOMMENDED:

One

SUPPLIES:

Information about the uncertainties of the instruments used in the vitamin C analysis activity (to be agreed upon in Science class and brought to math class).

OVERVIEW AND REMARKS:

You and the science teacher may wish to repeat this sort of exercise in the future whenever the two of you feel that it is appropriate.